EXERCISES Section 3.5 (68–78)

69. Each of 12 refrigerators of a certain type has been returned to a distributor because of an audible, high-pitched, oscillating noise when the refrigerator is running. Suppose that 7 of these refrigerators have a defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order, let X be the number among the first 6 examined that have a defective compressor. Compute the following:
    a. $P(X = 5)$
    b. $P(X \leq 4)$
    c. The probability that X exceeds its mean value by more than 1 standard deviation.
    d. Consider a large shipment of 400 refrigerators, of which 40 have defective compressors. If X is the number among 15 randomly selected refrigerators that have defective compressors, describe a less tedious way to calculate (at least approximately) $P(X \leq 5)$ than to use the hypergeometric pmf.

71. A geologist has collected 10 specimens of basaltic rock and 10 specimens of granite. The geologist instructs a laboratory assistant to randomly select 15 of the specimens for analysis.
    a. What is the pmf of the number of granite specimens selected for analysis?
    b. What is the probability that all specimens of one of the two types of rock are selected for analysis?
    c. What is the probability that the number of granite specimens selected for analysis is within 1 standard deviation of its mean value?

73. Twenty pairs of individuals playing in a bridge tournament have been seeded 1, . . . , 20. In the first part of the tournament, the 20 are randomly divided into 10 east–west pairs and 10 north–south pairs.
    a. What is the probability that x of the top 10 pairs end up playing east–west?
    b. What is the probability that all of the top five pairs end up playing the same direction?
    c. If there are $2n$ pairs, what is the pmf of X = the number among the top n pairs who end up playing east–west? What are $E(X)$ and $V(X)$?

75. Suppose that $p = P(\text{male birth}) = .5$. A couple wishes to have exactly two female children in their family. They will have children until this condition is fulfilled.
    a. What is the probability that the family has x male children?
    b. What is the probability that the family has four children?
    c. What is the probability that the family has at most four children?
    d. How many male children would you expect this family to have? How many children would you expect this family to have?

EXERCISES Section 3.6 (79–93)

79. Let X, the number of flaws on the surface of a randomly selected boiler of a certain type, have a Poisson distribution with parameter $\lambda = 5$. Use Appendix Table A.2 to compute the following probabilities:

a. $P(X \le 8)$ **b.** $P(X = 8)$ **c.** $P(9 \le X)$
d. $P(5 \le X \le 8)$ **e.** $P(5 < X < 8)$

81. Suppose that the number of drivers who travel between a particular origin and destination during a designated time period has a Poisson distribution with parameter $\lambda = 20$ (suggested in the article "Dynamic Ride Sharing: Theory and Practice," *J. of Transp. Engr.*, 1997: 308–312). What is the probability that the number of drivers will

a. Be at most 10?
b. Exceed 20?
c. Be between 10 and 20, inclusive? Be strictly between 10 and 20?
d. Be within 2 standard deviations of the mean value?

83. An article in the *Los Angeles Times* (Dec. 3, 1993) reports that 1 in 200 people carry the defective gene that causes inherited colon cancer. In a sample of 1000 individuals, what is the approximate distribution of the number who carry this gene? Use this distribution to calculate the approximate probability that

a. Between 5 and 8 (inclusive) carry the gene.
b. At least 8 carry the gene.

85. Suppose small aircraft arrive at a certain airport according to a Poisson process with rate $\alpha = 8$ per hour, so that the number of arrivals during a time period of t hours is a Poisson rv with parameter $\lambda = 8t$.

a. What is the probability that exactly 6 small aircraft arrive during a 1-hour period? At least 6? At least 10?
b. What are the expected value and standard deviation of the number of small aircraft that arrive during a 90-min period?
c. What is the probability that at least 20 small aircraft arrive during a $2\frac{1}{2}$-hour period? That at most 10 arrive during this period?

EXERCISES Section 4.1 (1–10)

1. Let X denote the amount of time for which a book on 2-hour reserve at a college library is checked out by a randomly selected student and suppose that X has density function

$$f(x) = \begin{cases} .5x & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the following probabilities:

a. $P(X \le 1)$
b. $P(.5 \le X \le 1.5)$
c. $P(1.5 < X)$

3. The error involved in making a certain measurement is a continuous rv X with pdf

$$f(x) = \begin{cases} .09375(4 - x^2) & -2 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a. Sketch the graph of $f(x)$.
b. Compute $P(X > 0)$.
c. Compute $P(-1 < X < 1)$.
d. Compute $P(X < -.5 \text{ or } X > .5)$.

5. A college professor never finishes his lecture before the end of the hour and always finishes his lectures within 2 min after the hour. Let X = the time that elapses between the end of the hour and the end of the lecture and suppose the pdf of X is

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the value of k and draw the corresponding density curve. [*Hint:* Total area under the graph of $f(x)$ is 1.]
b. What is the probability that the lecture ends within 1 min of the end of the hour?
c. What is the probability that the lecture continues beyond the hour for between 60 and 90 sec?
d. What is the probability that the lecture continues for at least 90 sec beyond the end of the hour?

EXERCISES Section 4.2 (11–27)

11. The cdf of checkout duration X as described in Exercise 1 is

$$F(x) = \begin{cases} x < 0 & \\ \frac{x^2}{4} & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

Use this to compute the following:

a. $P(X \le 1)$

b. $P(.5 \le X \le 1)$

c. $P(X > .5)$

d. The median checkout duration $\tilde{\mu}$ [solve $.5 = F(\tilde{\mu})$]

e. $F'(x)$ to obtain the density function $f(x)$

f. $E(X)$

g. $V(X)$ and σ_X

h. If the borrower is charged an amount $h(X) = X^2$ when checkout duration is X, compute the expected charge $E[h(X)]$.

15. Let X denote the amount of space occupied by an article placed in a 1-ft^3 packing container. The pdf of X is

$$f(x) = \begin{cases} 90x^8(1 - x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Graph the pdf. Then obtain the cdf of X and graph it.

b. What is $P(X \le .5)$ [i.e., $F(.5)$]?

c. Using the cdf from (a), what is $P(.25 < X \le .5)$? What is $P(.25 \le X \le .5)$?

d. What is the 75th percentile of the distribution?

e. Compute $E(X)$ and σ_X.

f. What is the probability that X is more than 1 standard deviation from its mean value?

EXERCISES Section 4.4 (59–71)

59. Let X = the time between two successive arrivals at the drive-up window of a local bank. If X has an exponential distribution with $\lambda = 1$ (which is identical to a standard gamma distribution with $\alpha = 1$), compute the following:

a. The expected time between two successive arrivals

b. The standard deviation of the time between successive arrivals

c. $P(X \le 4)$

d. $P(2 \le X \le 5)$

61. Extensive experience with fans of a certain type used in diesel engines has suggested that the exponential distribution provides a good model for time until failure. Suppose the mean time until failure is 25,000 hours. What is the probability that

a. A randomly selected fan will last at least 20,000 hours? At most 30,000 hours? Between 20,000 and 30,000 hours?

b. The lifetime of a fan exceeds the mean value by more than 2 standard deviations? More than 3 standard deviations?

EXERCISES Section 4.3 (28–58)

28. Let Z be a standard normal random variable and calculate the following probabilities, drawing pictures wherever appropriate.

a. $P(0 \le Z \le 2.17)$
b. $P(0 \le Z \le 1)$
c. $P(-2.50 \le Z \le 0)$
d. $P(-2.50 \le Z \le 2.50)$
e. $P(Z \le 1.37)$
f. $P(-1.75 \le Z)$
g. $P(-1.50 \le Z \le 2.00)$
h. $P(1.37 \le Z \le 2.50)$
i. $P(1.50 \le Z)$
j. $P(|Z| \le 2.50)$

31. Determine z_α for the following:
a. $\alpha = .0055$ **b.** $\alpha = .09$
c. $\alpha = .663$

33. Suppose the force acting on a column that helps to support a building is normally distributed with mean 15.0 kips and standard deviation 1.25 kips. What is the probability that the force

a. Is at most 18 kips?
b. Is between 10 and 12 kips?
c. Differs from 15.0 kips by at most 1.5 standard deviations?

35. Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$, as suggested in the article "Simulating a Harvester-Forwarder Softwood Thinning" (*Forest Products J.*, May 1997: 36–41).

a. What is the probability that the diameter of a randomly selected tree will be at least 10 in.? Will exceed 10 in.?
b. What is the probability that the diameter of a randomly selected tree will exceed 20 in.?
c. What is the probability that the diameter of a randomly selected tree will be between 5 and 10 in.?
d. What value c is such that the interval $(8.8 - c, 8.8 + c)$ includes 98% of all diameter values?
e. If four trees are independently selected, what is the probability that at least one has a diameter exceeding 10 in.?

37. Suppose that blood chloride concentration (mmol/L) has a normal distribution with mean 104 and standard deviation 5 (information in the article "Mathematical Model of Chloride Concentration in Human Blood," *J. of Med. Engr. and Tech.*, 2006: 25–30, including a normal probability plot as described in Section 4.6, supports this assumption).

a. What is the probability that chloride concentration equals 105? Is less than 105? Is at most 105?
b. What is the probability that chloride concentration differs from the mean by more than 1 standard deviation? Does this probability depend on the values of μ and σ?
c. How would you characterize the most extreme .1% of chloride concentration values?

47. The weight distribution of parcels sent in a certain manner is normal with mean value 12 lb and standard deviation 3.5 lb. The parcel service wishes to establish a weight value c beyond which there will be a surcharge. What value of c is such that 99% of all parcels are at least 1 lb under the surcharge weight?

EXERCISES Section 1.3 (33–43)

33. The article "The Pedaling Technique of Elite Endurance Cyclists" (*Int. J. of Sport Biomechanics,* 1991: 29–53) reported the accompanying data on single-leg power at a high workload:

244	191	160	187	180	176	174
205	211	183	211	180	194	200

a. Calculate and interpret the sample mean and median.

b. Suppose that the first observation had been 204 rather than 244. How would the mean and median change?

c. Calculate a trimmed mean by eliminating the smallest and largest sample observations. What is the corresponding trimming percentage?

d. The article also reported values of single-leg power for a low workload. The sample mean for $n = 13$ observations was $\bar{x} = 119.8$ (actually 119.7692), and the 14th observation, somewhat of an outlier, was 159. What is the value of $\bar{x}$ for the entire sample?

37. The article "Snow Cover and Temperature Relationships in North America and Eurasia" (*J. Climate and Applied Meteorology,* 1983: 460–469) used statistical techniques to relate the amount of snow cover on each continent to average

What would you report as a representative, or typical, value of October snow cover for this period, and what prompted your choice?

39. The propagation of fatigue cracks in various aircraft parts has been the subject of extensive study in recent years. The accompanying data consists of propagation lives (flight hours/10^4) to reach a given crack size in fastener holes intended for use in military aircraft ("Statistical Crack Propagation in Fastener Holes under Spectrum Loading," *J. Aircraft,* 1983: 1028–1032):

.736	.863	.865	.913	.915	.937	.983	1.007
1.011	1.064	1.109	1.132	1.140	1.153	1.253	1.394

a. Compute and compare the values of the sample mean and median.

b. By how much could the largest sample observation be decreased without affecting the value of the median?

EXERCISES Section 1.4 (44–61)

47. Calculate and interpret the values of the sample median, sample mean, and sample standard deviation for the following observations on fracture strength (MPa, read from a graph in "Heat-Resistant Active Brazing of Silicon Nitride: Mechanical Evaluation of Braze Joints," *Welding J.,* August, 1997):

87	93	96	98	105	114	128	131	142	168

49. A study of the relationship between age and various visual functions (such as acuity and depth perception) reported the following observations on area of scleral lamina (mm^2) from human optic nerve heads ("Morphometry of Nerve Fiber Bundle Pores in the Optic Nerve Head of the Human," *Experimental Eye Research,* 1988: 559–568):

2.75	2.62	2.74	3.85	2.34	2.74	3.93	4.21	3.88
4.33	3.46	4.52	2.43	3.65	2.78	3.56	3.01	

a. Calculate $\sum x_i$ and $\sum x_i^2$.

b. Use the values calculated in part (a) to compute the sample variance s^2 and then the sample standard deviation s.

51. The article "A Thin-Film Oxygen Uptake Test for the Evaluation of Automotive Crankcase Lubricants" (*Lubric. Engr.,* 1984: 75–83) reported the following data on oxidation-induction time (min) for various commercial oils:

87	103	130	160	180	195	132	145	211	105	145
153	152	138	87	99	93	119	129			

a. Calculate the sample variance and standard deviation.

b. If the observations were reexpressed in hours, what would be the resulting values of the sample variance and sample standard deviation? Answer without actually performing the reexpression.

EXERCISES Section 6.1 (1–19)

1. The accompanying data on flexural strength (MPa) for concrete beams of a certain type was introduced in Example 1.2.

5.9	7.2	7.3	6.3	8.1	6.8	7.0
7.6	6.8	6.5	7.0	6.3	7.9	9.0
8.2	8.7	7.8	9.7	7.4	7.7	9.7
7.8	7.7	11.6	11.3	11.8	10.7	

a. Calculate a point estimate of the mean value of strength for the conceptual population of all beams manufactured in this fashion, and state which estimator you used. [*Hint:* $\sum x_i = 219.8$.]

b. Calculate a point estimate of the strength value that separates the weakest 50% of all such beams from the strongest 50%, and state which estimator you used.

c. Calculate and interpret a point estimate of the population standard deviation σ. Which estimator did you use? [*Hint:* $\sum x_i^2 = 1860.94$.]

d. Calculate a point estimate of the proportion of all such beams whose flexural strength exceeds 10 MPa. [*Hint:* Think of an observation as a "success" if it exceeds 10.]

e. Calculate a point estimate of the population coefficient of variation σ/μ, and state which estimator you used.

3. Consider the following sample of observations on coating thickness for low-viscosity paint ("Achieving a Target Value for a Manufacturing Process: A Case Study," *J. of Quality Technology,* 1992: 22–26):

.83	.88	.88	1.04	1.09	1.12	1.29	1.31
1.48	1.49	1.59	1.62	1.65	1.71	1.76	1.83

Assume that the distribution of coating thickness is normal (a normal probability plot strongly supports this assumption).

a. Calculate a point estimate of the mean value of coating thickness, and state which estimator you used.

b. Calculate a point estimate of the median of the coating thickness distribution, and state which estimator you used.

c. Calculate a point estimate of the value that separates the largest 10% of all values in the thickness distribution from the remaining 90%, and state which estimator you used. [*Hint:* Express what you are trying to estimate in terms of μ and σ.]

d. Estimate $P(X < 1.5)$, i.e., the proportion of all thickness values less than 1.5. [*Hint:* If you knew the values of μ and σ, you could calculate this probability. These values are not available, but they can be estimated.]

e. What is the estimated standard error of the estimator that you used in part (b)?

5. As an example of a situation in which several different statistics could reasonably be used to calculate a point estimate, consider a population of N invoices. Associated with each invoice is its "book value," the recorded amount of that invoice. Let T denote the total book value, a known amount. Some of these book values are erroneous. An audit will be carried out by randomly selecting n invoices and determining the audited (correct) value for each one. Suppose that the sample gives the following results (in dollars).

	Invoice				
	1	**2**	**3**	**4**	**5**
Book value	300	720	526	200	127
Audited value	300	520	526	200	157
Error	0	200	0	0	−30

Let

$\overline{Y}$ = sample mean book value

$\overline{X}$ = sample mean audited value

$\overline{D}$ = sample mean error

Propose three different statistics for estimating the total audited (i.e., correct) value—one involving just N and $\overline{X}$, another involving T, N, and $\overline{D}$, and the last involving T and $\overline{X}/\overline{Y}$. If $N = 5000$ and $T = 1{,}761{,}300$, calculate the three corresponding point estimates. (The article "Statistical Models and Analysis in Auditing," *Statistical Science,* 1989: 2–33). discusses properties of these estimators.)

EXERCISES Section 6.2 (20–30)

20. A random sample of n bike helmets manufactured by a certain company is selected. Let X = the number among the n that are flawed, and let $p = P(\text{flawed})$. Assume that only X is observed, rather than the sequence of S's and F's.
   a. Derive the maximum likelihood estimator of p. If $n = 20$ and $x = 3$, what is the estimate?
   b. Is the estimator of part (a) unbiased?
   c. If $n = 20$ and $x = 3$, what is the mle of the probability $(1 - p)^5$ that none of the next five helmets examined is flawed?

EXERCISES Section 5.3 (37–45)

37. A particular brand of dishwasher soap is sold in three sizes: 25 oz, 40 oz, and 65 oz. Twenty percent of all purchasers select a 25-oz box, 50% select a 40-oz box, and the remaining 30% choose a 65-oz box. Let X_1 and X_2 denote the package sizes selected by two independently selected purchasers.
   a. Determine the sampling distribution of $\overline{X}$, calculate $E(\overline{X})$, and compare to μ.
   b. Determine the sampling distribution of the sample variance S^2, calculate $E(S^2)$, and compare to σ^2.

EXERCISES Section 5.4 (46–57)

46. The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviation .04 cm.
   a. If $\overline{X}$ is the sample mean diameter for a random sample of $n = 16$ rings, where is the sampling distribution of $\overline{X}$ centered, and what is the standard deviation of the $\overline{X}$ distribution?
   b. Answer the questions posed in part (a) for a sample size of $n = 64$ rings.
   c. For which of the two random samples, the one of part (a) or the one of part (b), is $\overline{X}$ more likely to be within .01 cm of 12 cm? Explain your reasoning.

47. Refer to Exercise 46. Suppose the distribution of diameter is normal.
   a. Calculate $P(11.99 \le \overline{X} \le 12.01)$ when $n = 16$.
   b. How likely is it that the sample mean diameter exceeds 12.01 when $n = 25$?

49. There are 40 students in an elementary statistics class. On the basis of years of experience, the instructor knows that the time needed to grade a randomly chosen first examination paper is a random variable with an expected value of 6 min and a standard deviation of 6 min.
   a. If grading times are independent and the instructor begins grading at 6:50 P.M. and grades continuously, what is the (approximate) probability that he is through grading before the 11:00 P.M. TV news begins?
   b. If the sports report begins at 11:10, what is the probability that he misses part of the report if he waits until grading is done before turning on the TV?

53. Rockwell hardness of pins of a certain type is known to have a mean value of 50 and a standard deviation of 1.2.
   a. If the distribution is normal, what is the probability that the sample mean hardness for a random sample of 9 pins is at least 51?
   b. What is the (approximate) probability that the sample mean hardness for a random sample of 40 pins is at least 51?

59. Let X_1, X_2, and X_3 represent the times necessary to perform three successive repair tasks at a certain service facility. Suppose they are independent, normal rv's with expected values μ_1, μ_2, and μ_3 and variances σ_1^2, σ_2^2, and σ_3^2, respectively.

a. If $\mu = \mu_2 = \mu_3 = 60$ and $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 15$, calculate $P(X_1 + X_2 + X_3 \le 200)$. What is $P(150 \le X_1 + X_2 + X_3 \le 200)$?

b. Using the μ_is and σ_is given in part (a), calculate $P(55 \le \overline{X})$ and $P(58 \le \overline{X} \le 62)$.

c. Using the μ_is and σ_is given in part (a), calculate $P(-10 \le X_1 - .5X_2 - .5X_3 \le 5)$.

d. If $\mu_1 = 40$, $\mu_2 = 50$, $\mu_3 = 60$, $\sigma_1^2 = 10$, $\sigma_2^2 = 12$, and $\sigma_3^2 = 14$, calculate $P(X_1 + X_2 + X_3 \le 160)$ and $P(X_1 + X_2 \ge 2X_3)$.

65. Suppose that when the pH of a certain chemical compound is 5.00, the pH measured by a randomly selected beginning chemistry student is a random variable with mean 5.00 and standard deviation .2. A large batch of the compound is subdivided and a sample given to each student in a morning lab and each student in an afternoon lab. Let $\overline{X}$ = the average pH as determined by the morning students and $\overline{Y}$ = the average pH as determined by the afternoon students.

a. If pH is a normal variable and there are 25 students in each lab, compute $P(-.1 \le \overline{X} - \overline{Y} \le .1)$. [*Hint:* $\overline{X} - \overline{Y}$ is a linear combination of normal variables, so is normally distributed. Compute $\mu_{\overline{X}-\overline{Y}}$ and $\sigma_{\overline{X}-\overline{Y}}$.]

b. If there are 36 students in each lab, but pH determinations are not assumed normal, calculate (approximately) $P(-.1 \le \overline{X} - \overline{Y} \le .1)$.