Topic #1: Introduction to Functions

Objectives

- Define *function*
- Determine if a situation represents a functional relationship
- Identify independent and dependent variables
- Identify the domain & range of a function given a formula or graph
- Express functions using (1) words, (2) graphs, (3) tables, and (4) formulas with appropriate notation
- Evaluate functions
- Explain why the VLT determines if a graph represents a function
- Identify the graphs of elementary functions
- Describe the behavior of graphs using correct terminology
- Calculate the average rate of change of a function over a set interval
- Solve basic equations analytically and graphically

1) Introduce yourself to another student in the class. Record the following information for that other student.

   First name: _____ Hometown: _____

   Intended major: _____ Which zoo animal is the coolest? _____

   On a scale from 1 (hate it) to 10 (love it), the student's feelings towards math: _____

   A movie, tv show, video game, book, or activity the student is embarrassed to admit enjoying: _____

2) We'll have you introduce that student to the rest of the class by sharing the above information. On the board, we'll record some of this data. Then, we'll review the syllabus and figure out what we're going to learn in this class.

3) Over the next 2 days, humans will generate more information than we did from the dawn of civilization until 2003.* Think about all the information you've already already generated at St. Ambrose…
   - demographic data (name, gender, race, age, address, phone number, email address, student ID)
   - achievement data (ACT scores, high school GPA, course grades)
   - enrollment data (the courses you're taking, your major, housing information)
   - financial data (financial aid, scholarships, things you buy with your ID card)
   - attitudes/affective data (responses to the MAP-Works or other surveys)

   Combine that with all the other data you generated just today (email or text messages you sent, websites you visited, photos/videos you created/uploaded, items you bought) and you can see that we're generating lots of data that could be used for all sorts of purposes.

   Most of you, after graduating, will go on to careers or graduate programs that will require you to have the skills to store, access, analyze, and understand data to make decisions.

   In this class, we will learn how to use *elementary functions* to construct *mathematical models* – mathematical representations of data – to gain insight and solve problems.

* http://techcrunch.com/2010/08/04/schmidt-data/

4) Your grade in this class is a function of the amount of effort you put into the class.
The fine you pay for a speeding ticket is a function of how fast you were driving and the speed limit in the area.

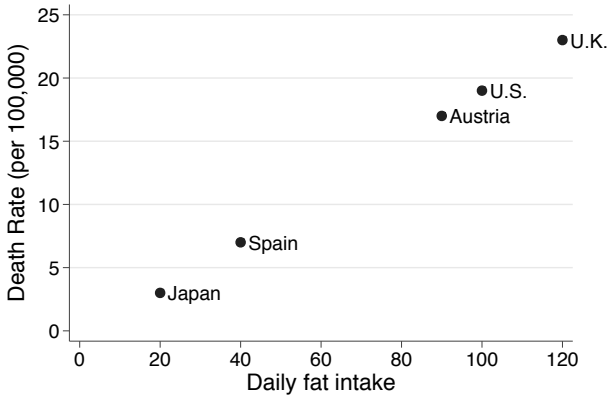
In these statements, what does the phrase "is a function of" mean to you?

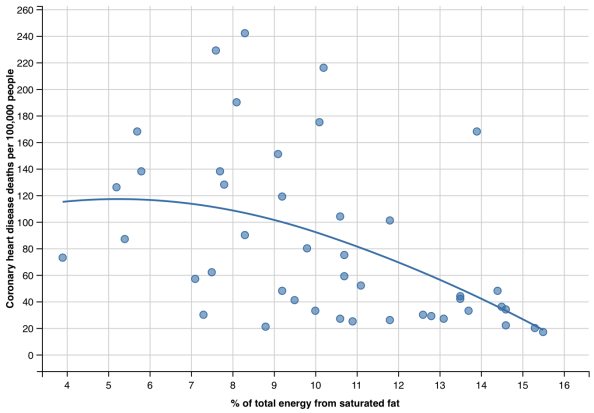
5) Suppose we're interested in the relationship between diet and health. Google led me to the following datasets showing the relationship between the amount of fat consumed and the death rates for various countries:

Country*	Daily Fat intake (grams)	Death rate (per 100,000)
Japan	20	3
Spain	40	7
Austria	90	17
U.S.	100	19
U.K.	120	23

European Nation**	% of daily energy from saturated fats	Death rate (per 100,000) in 2005
Albania	9.2	48
Armenia	7.3	7
…	…	…
Romania	8.3	90
Russia	8.3	242
…	…	…
Uzbekistan	9.2	119

(Data from 40 other nations not shown in this table)





What does each dataset tell us about the relationship between fat and death rates? Would you say that a nation's death rate is a function of the amount of fat consumed by its citizens?

* Gordon, S.P. & Gordon, F.S. (2010). Functions, Data and Models: An Applied Approach to College Algebra

** European cardiovascular disease statistics, 2008 edition.
http://www.herzstiftung.ch/uploads/media/European_cardiovascular_disease_statistics_2008.pdf

In math, a **function** is a relationship
between a set of **inputs** (independent variables)
and a set of **outputs** (dependent variables)
such that **each input is related to exactly one output**.

We say **the dependent variable is a function of the independent variable**

6) For each of the following scenarios, identify the independent and dependent variables. If the scenario represents a function, write out the function in a sentence. If the scenario does not represent a function, explain why.

Country*	Daily Fat intake (grams)	Death rate (per 100,000)
Japan	20	3
Spain	40	7
Austria	90	17
U.S.	100	19
U.K.	120	23

Independent variable: _____

Dependent variable: _____

Function? _____

_____

_____

European Nation**	% of daily energy from saturated fats	Death rate (per 100,000) in 2005
Albania	9.2	48
Armenia	7.3	7
…	…	…
Romania	8.3	90
Russia	8.3	242
…	…	…
Uzbekistan	9.2	119

(Data from 40 other nations not shown in this table)

Independent variable: _____

Dependent variable: _____

Function? _____

_____

_____

Year	Golf courses in U.S.
2000	15489
2002	15827
2004	16057
2006	15990
2008	15979
2009	15979
2010	15890

Independent variable: _____

Dependent variable: _____

Function? _____

_____

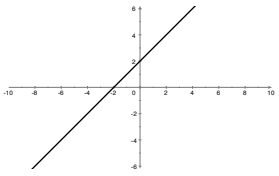
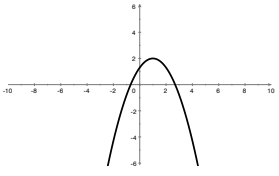
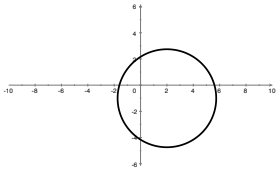
_____

7) Writing out all those functions quickly gets boring. That's one reason why we tend to use functional notation.

The dependent variable	is	a function of	the independent variable
output	is	a function of	input
y	=	f(	x)

y = f(x)

Read each scenario and circle the relationships that represent functions. Explain why each one is or isn't a function.

Scenario	Circle the function(s)	
a) **I = an individual** and **M = that person's mother**.	**I = f(M)**	**M = f(I)**
b) **H = your height** and **A = your age**.	**H = f(A)**	**A = f(H)**
c) **P = product for sale** and **$ = price**.	**P = f($)**	**$ = f(P)**
d) **C = circumference of a circle** and **R = radius**.	**C = f(R)**	**R = f(C)**
e) **y = 3x + 2**.	**y = f(x)**	**x = f(y)**
f) **$y = x^2$**	**y = f(x)**	**x = f(y)**
g) **$y = x^2$.**	**y = f(x)**	**x = f(y)**
h) 	**y = f(x)**	**x = f(y)**
i) 	**y = f(x)**	**x = f(y)**
j) 	**y = f(x)**	**x = f(y)**

8) Can you think of a way to quickly determine if a graph represents a function? Explain why your method works.

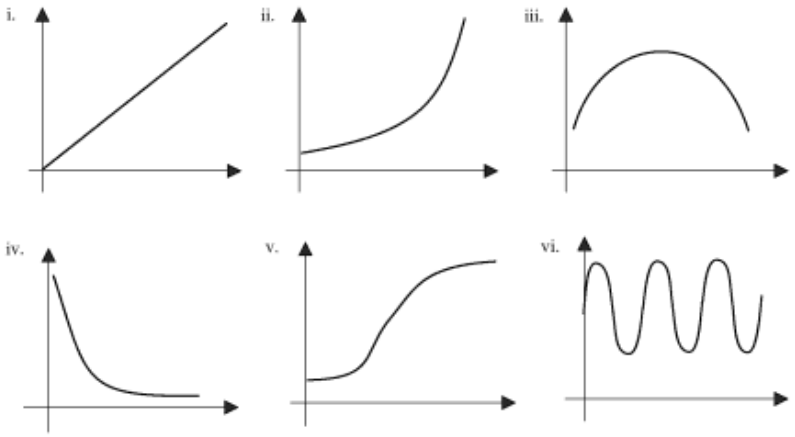
9) Determine which graph corresponds with each of the following scenarios and identify the independent variable.

a) Height of a thrown football = f(time)

b) Number infected = f(days since outbreak)

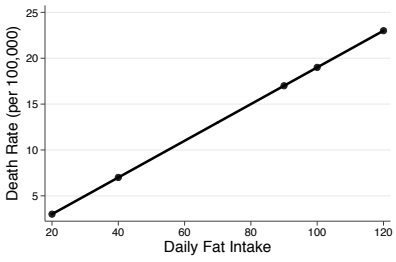
c) How much you like math = f(days in this class)

d) Distance driven at constant speed = f(time)



10) As you've seen, functions can be represented in words, tables, graphs, and formulas. To better understand a function, we'll often use multiple representations.

Looking again at the relationship between fat intake and death rates, you can see that all the data fit perfectly on a line. It turns out the formula for that line is: $y = -1 + 0.2x$ or $d = -1 + 0.2f$



a) The average daily fat intake in Mexico is 23 grams. Based on our formula, what's the death rate in Mexico?

b) The average daily fat intake in Denmark is 135 grams. Based on our formula, what's the death rate in Denmark?

c) Suppose I know a country with a death rate of 10. What would you estimate for the average daily fat intake of that country?

11) Let's look one last time at some of saturated fat related data for 10 European nations. For these 10 nations, D = f(S). Evaluate the following:

f(5.2) = _____

f(15.4) = _____

f(_____) = 138

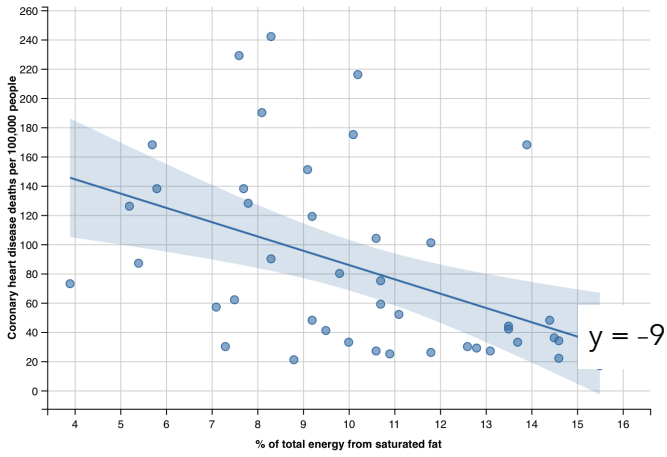
European Nation**	% of daily energy from saturated fats	Death rate (per 100,000) in 2005
Bosnia	3.9	73
Georgia	5.2	126
Tajikistan	5.4	87
Azerbaijan	5.7	168
Moldova	5.8	138
Belgium	14.5	36
Iceland	14.6	34
Netherlands	14.7	22
Switzerland	15.3	20
France	15.5	17

11) Even though the data for all 40 nations do not fall on a line (or even represent a functional relationship), we can find the line that **best** fits the data. That line, along with its formula, is displayed below. Evaluate the following:

f(5.2) = _____

f(15.4) = _____

f(_____) = 138



12) Throughout this semester, we'll often find it useful to identify the domain and range of a function.

> The **domain** of a function refers to the set of **all possible values for the input** (independent variable)
> The **range** of a function refers to the set of **all possible values for the output** (dependent variable)

Given f(x) = 3x + 2, evaluate the following:

f(3) = _____ f(💩) = _____

Domain of f(x) = _____

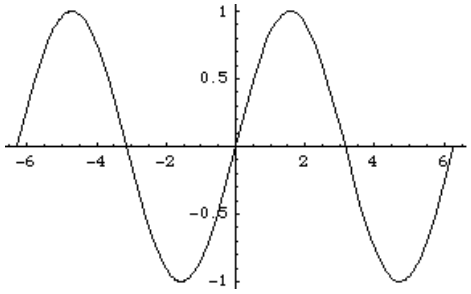
13) Identify the domain and range of the following:

Country	Daily Fat intake (grams)	Death rate (per 100,000)
Japan	20	3
Spain	40	7
Austria	90	17
U.S.	100	19
U.K.	120	23

Given death rate is a function of daily fat intake...

Domain: _____

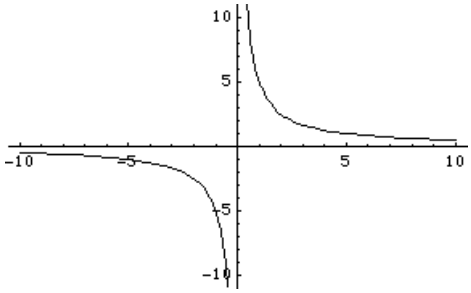
Range: _____



Given y = f(x)...

Domain: _____

Range: _____



Given y = f(x)...

Domain: _____

Range: _____

Given $f(x) = \sqrt{x-2}$

Domain: _____

Given $f(x) = \dfrac{1}{x-2}$

Domain: _____

14) Suppose the cost, C, to rent a car is modeled by the function $C(m) = 35 + 0.25m$, where m = miles driven.

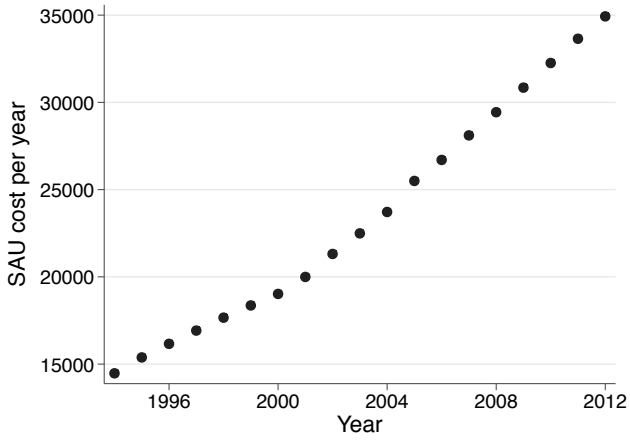
What does f(10) = 37.5 represent?

What are reasonable values for the domain and range in this situation?

Before next class, complete the following…

15) Think about the data we collected in class. For students in this class, is a student's favorite zoo animal a function of their major? Briefly explain.

16) As you're probably aware, the cost of attending college keeps increasing. The cost (C) to attend St. Ambrose each year (y) from '94-'12 is displayed below. As it turns out, the cost can be modeled by: $C(y) = (14447)1.05^{(y-1994)}$



Year	Cost
1994	14470
1995	15380
1996	16160
1997	16920
1998	17660
1999	18360
2000	19020
2001	19994
2002	21310
2003	22490

Year	Cost
2004	23716
2005	25496
2006	26700
2007	28106
2008	29434
2009	30844
2010	32255
2011	33644
2012	34926

Source: http://www.sau.edu/Institutional_Research/Reports.html

a) Is the cost of attending SAU a function of the year? If so, identify the domain and range.

b) How much did the cost of attending SAU change from 1994 to 2012? What is the average change per year?

c) Using either the graph or the formula, estimate the year it will cost $50,000 to attend St. Ambrose?

d) According to our model, what would C(2014) represent? Use the formula to calculate C(2014).

e) Is this function increasing or decreasing? Is it concave up or concave down? Does our function have a constant rate of change?

17) For 3 different vending machines, pushing a button (B) will yield a snack (S).

Vending Machine #1

B	S
1	M&Ms
2	Pretzels
3	Dried Fruit
4	Hershey's
5	Cookies
6	Snickers

Vending Machine #2

B	S
1	M&Ms or Dried Fruit
2	Pretzels or Hershey's
3	Cookies or Snickers

Vending Machine #3

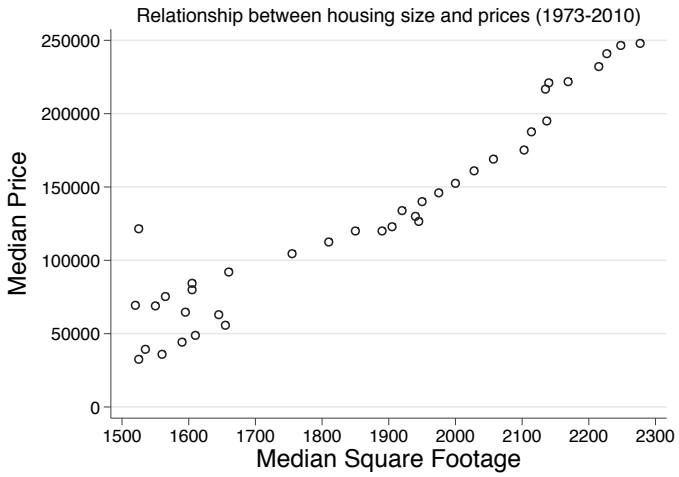
B	S
1	M&Ms
2	M&Ms
3	Pretzels
4	Dried Fruit
5	Hershey's
6	Hershey's
7	Cookies
8	Snickers
9	Snickers

a) For which machine(s), if any, is S a function of B? In other words, which functions represent S = f(B)?

b) For which machine(s), if any, is B a function of S? In other words, which functions represent B = f(S)?

17) As the following graph shows, the price of a new home built in the U.S. is a function of the size of the house*. By hand, sketch a curve (or straight line) that you think best models the relationship between the size and cost of a new home. Then, use your curve to estimate the following:

* http://www.census.gov/const/uspriceann.pdf



a) Estimated cost of a 1700-square foot home = _____

b) Estimated size of a home that costs $200,000 = _____