Activity #11: Rational Functions

(student learning outcomes listed in syllabus)

1) The following table displays how much 12 pharmaceutical companies spent to develop each drug they sell:

Company	R&D cost per drug ($Mil)	Company	R&D cost per drug ($Mil)
AstraZeneca	11,790.93	Eli Lilly & Co.	11,790.93
GlaxoSmithKline	8,170.81	Abbott Laboratories	8,170.81
Sanofi	7,909.26	Merck & Co Inc	7,909.26
Roche Holdings, Inc.	7,803.77	Bristol-Meyers Squibb	7,803.77
Pfizer	7,727.03	Novartis AG	7,727.03
Johnson & Johnson	5,885.65	Amgen	5,885.65

Source: http://www.forbes.com/sites/matthewherper/2012/02/10/the-truly-staggering-cost-of-inventing-new-drugs/

One of the drugs that Amgen sells is *Aranesp*, which is used to treat anemia (commonly due to renal failure or chemotherapy). Suppose it cost Amgen $5,885,650,000 to develop Aranesp. Further, suppose it costs Amgen $40 to produce each dose of the drug.

Write a formula to model Amgen's total cost as a function of the number of doses of Aranesp it produces. Explain what the slope and y-intercept represent.

2) Suppose Amgen produces 100,000,000 doses of Aranesp. Write a formula to model Amgen's revenues as a function of the price they charge per dose. How high must they set the price to break even?

3) If Amgen produces 100,000,000 doses of Aranesp, how high must they set the price to break even? Write a formula to model Amgen's profits as a function of the number of doses they sell.

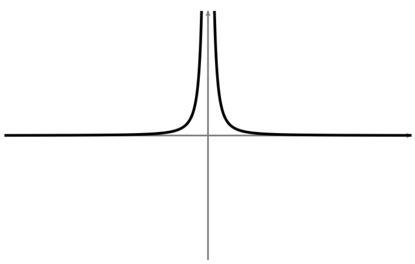
4) Sketch the graph of this function below. How much must Amgen charge per dose if they produce 100 doses, 100,000 doses, or 500,000,000 doses? What are the domain and range of this function?

5) Can Amgen ever charge $40 or less for a dose of *Aranesp*?

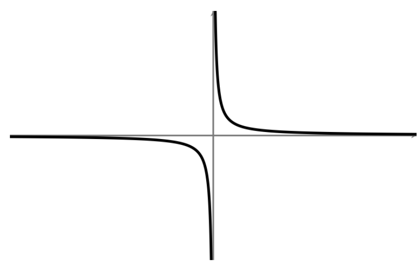
A **rational function** is a ratio of two polynomials:

$$r(x) = \frac{f(x)}{g(x)}, \text{ where } g(x) \neq 0$$

6) Let's look at a couple simple rational functions: $r(x) = \frac{1}{x}$ and $s(x) = \frac{1}{x^2}$. The graphs are displayed below.



$$r(x) = \frac{1}{x}$$



$$s(x) = \frac{1}{x^2}$$

Identify the domain and range of each function. Describe the behavior of each function as x approaches zero. Describe the behavior of each function as x gets extremely large.

With rational functions, we're often interested in locating the vertical asymptotes, horizontal asymptotes, and zeros. Using the following definitions, locate the asymptotes of r(x) and s(x).

For a rational function, r(x):

If $r(x) \to \infty$ as $x \to a$, then $x = a$ is a vertical asymptote.

If $r(x) \to a$ as $x \to \infty$, then $y = a$ is a horizontal asymptote.

We could also write:

If $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$, then $x = a$ is a vertical asymptote.

If $\lim_{x \to -\infty} f(x) = a$ or $\lim_{x \to \infty} f(x) = a$, then $y = a$ is a horizontal asymptote.

7) To find the vertical asymptotes of a rational function, we set the denominator equal to zero and solve for x. Graph the following rational functions and find the vertical asymptotes.

$$f(x) = \frac{3x+7}{x+2}$$

$$g(x) = \frac{5+2x^2}{2-x-x^2}$$

$$h(x) = \frac{x-2}{x^2-4}$$

8) That last example demonstrates that setting the denominator equal to zero and solving for x will not always find vertical asymptotes. This method sometimes finds holes in our graph. A hole might occur in the graph if the input causes both the numerator and denominator to equal zero.

To find a horizontal asymptotes, we need to determine what happens to the function as the input grows extremely large. In other words, we need to find how high (or low) a function intends to go as x approaches infinity.

We can classify rational functions into three categories and generalize methods to find horizontal asymptotes.

Category #1: The degree of the numerator is **less than** the degree of the denominator.

Category #2: The degree of the numerator is **equal to** the degree of the denominator.

Category #3: The degree of the numerator is **greater than** the degree of the denominator.

Below, write one rational function from each category.

9) Let's look at rational functions in which the degree of the numerator is **less than** the degree of the denominator.

$$a(x) = \frac{x-2}{x^2-4}$$

$$b(x) = \frac{3x^2+2x-4}{6x^3+3}$$

$$c(x) = \frac{14x^{167}+37x^{32}+12x^8+6x^2+24}{3x^{170}+2x^{145}-1}$$

What happens to each function as x gets extremely large? Can we create a general method for finding horizontal asymptotes for this type of rational function?

10) Let's look at rational functions in which the degree of the numerator is **equal to** the degree of the denominator.

$$a(x) = \frac{3x+7}{x+2}$$

$$b(x) = \frac{3x^2+2x-4}{6x^2+3}$$

$$c(x) = \frac{14x^{167}+37x^{32}+12x^8+6x^2+24}{3x^{167}+2x^{145}-1}$$

What happens to each function as x gets extremely large? Can we create a general method for finding horizontal asymptotes for this type of rational function?

11) Let's look at rational functions in which the degree of the numerator is **greater than** the degree of the denominator.

$$a(x) = \frac{3x^2 + 7}{x + 2}$$

$$b(x) = \frac{3x^4 + 2x - 4}{6x^2 + 3}$$

$$c(x) = \frac{14x^{170} + 37x^{32} + 12x^8 + 6x^2 + 24}{3x^{167} + 2x^{145} - 1}$$

What happens to each function as x gets extremely large? Can we create a general method for finding horizontal asymptotes for this type of rational function?

12) Have another student in this class find the horizontal asymptotes for the rational functions you wrote on question #8.

13) Use what you know about transformations and rational functions to sketch the graph of $f(x) = \frac{3x + 7}{x + 2} = \frac{1}{x + 2} + 3$