Activity #15: Inverse Trigonometric Functions (student learning outcomes listed in syllabus)

Thus far in this unit, we've learned about basic trigonometric functions (sine, cosine, tangent, secant, cosecant, cotangent) and how to model data by modifying the amplitude, period, and midline of sinusoidal functions:

$$f(x) = A\sin B(x-h)+k \qquad\qquad f(x) = A\cos B(x-h)+k$$

In this activity, we'll learn how to solve trigonometric equations using inverse trigonometric functions.

1) Last time, we derived the following formula to model the population of rabbits at a national park as a function of the month:

$$R(m) = -5000\cos\left(\frac{\pi}{6}m\right)+10{,}000$$

Use your calculator to determine when the rabbit population will equal 12,000. How often will this happen?

2) Let's see if we can solve this analytically. Explain what is happening at each step:

$$12000 = -5000\cos\left(\frac{\pi}{6}m\right)+10{,}000$$

$$2000 = -5000\cos\left(\frac{\pi}{6}m\right)$$

$$-0.4 = \cos\left(\frac{\pi}{6}m\right)$$

$$\cos^{-1}(-0.4) = \cos^{-1}\left(\cos\left(\frac{\pi}{6}m\right)\right)$$

$$\cos^{-1}(-0.4) = \frac{\pi}{6}m$$

$$\frac{6}{\pi}\cos^{-1}(-0.4) = m \approx 3.786$$

3) Did this technique fully answer our question? How do we get the other answers?

4) Is this a valid technique to use? What does $\cos^{-1}(x)$ represent? Does the cosine function have an inverse?

5) Graph the arccosine function, $\cos^{-1}(x)$, on your calculator and sketch it below. Identify the domain and range.

6) Evaluate the following and briefly explain what they represent:

a. $\cos^{-1}(0)$

b. $\cos^{-1}(1)$

c. $\cos^{-1}(2)$

c. $\cos(\cos^{-1}(0))$

7) Your height on a Ferris Wheel is given by $h(t) = 250 + 250\sin\left(\frac{\pi}{10}t - \frac{\pi}{2}\right)$. How much time do you spend above 400 feet if your ride lasts 20 minutes?

8) Solve other problems from previous activity. Briefly explain limits, differentiation, antidifferentiation.