Activity #1: Composition of functions; Inverse functions (student learning outcomes listed in syllabus)

1) According to the University of Nebraska, each step you take burns 0.05 calories. You know your hero Jefferson Perez, the 1996 Olympic gold medalist and 3-time world record holder in the 20k race walk, took 186 steps per minute as he completed the event in 77.35 minutes. How many calories did Jefferson burn during this race? If he could keep up this pace, how far would he need to race walk to burn-off a Hardee's two-thirds pound Thickburger (1417 calories), medium fries (520 calories), and diet soda (0 calories)?

Sources: http://www.livestrong.com/article/72931-calculate-calories-burned-pedometer/
http://www.eracewalk.com/Tech.htm
http://www.dietfacts.com/html/nutrition-facts/hardees-2-3-lb-monster-thickburger-two-1-3-lb-patties-of-charbroiled-angus-beef-three-slice43069.htm

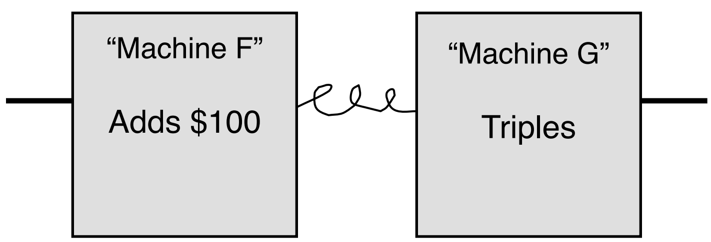
2) On your way to class today, you stumble across a magic lamp. As you rub the lamp, a genie appears and grants you 3 wishes. The only catch is he gives you 30 seconds to make all 3 wishes.

Under pressure, you make your first wish for an "automatic money machine." The genie, obviously confused because he's never heard of such a thing, grants your wish as an ATM-like machine appears. The genie tells you if you insert any money into this machine, it will give you back an extra $100. Oh, and you have 20 seconds left.

Realizing it will take too long to get rich from this machine, you wish for a second machine that will automatically triple any money you insert. The genie, disappointed by your lack of creativity, grants this wish and tells you there are 10 seconds left.

Your mind races. Should you wish for a cure for cancer, a super power, or for more wishes? No. You've made two wishes for money machines and you're going to go all-in. You tell the genie that you wish for an adapter to hook the machines together.

The genie disappears back into the magic lamp and you're left with the following:



It looks like you can put your money into either side of the machines.

Suppose the machines are not connected by the adapter. Write out formulas for machines F and G, showing what happens when you insert $x into each machine.

Suppose the machines are connected, so that whatever money you insert goes through both machines. Write out formulas showing what happens when you insert $x into either the left side or the right side of the two machines.

3) We've just gone through two examples of composition of functions. Complete the following table showing f(g(x)) and g(f(x)).

x	f(x)	g(x)	f(g(x))	g(f(x))
0	1	2	2	______
1	3	0	______	0
2	______	1	______	______
3	2	______	______	______

4) Find the following composite functions.

Given: $f(x) = x^2 - 1$

$g(x) = \sqrt{x}$

$h(x) = 3x + 2$

$i(x) = \dfrac{x+1}{x-1}$

f(g(x)) = ______________________________

g(f(x)) = ______________________________

h(i(x)) = ______________________________

i(f(x)) = ______________________________

f(g(h(x))) = ______________________________

i(h(g(f(x)))) = ______________________________

5) Define *function*, *domain*, and *range*. How can we tell if a graph represents a function? Why does this work?

6) My phone gives me the temperature in Celsius and I'm too lazy to change the settings. I do remember, though, that 0 Celsius and 32 Fahrenheit are the freezing points for water. Likewise, 100 Celsius corresponds with 212 Fahrenheit.

Use this information to find a formula expressing degrees Celsius as a function of degrees Fahrenheit.

7) Using the function you just found, convert 14 degrees Celsius to Fahrenheit.

8) Suppose we want to convert several temperatures from Celsius to Fahrenheit. Rather than doing what we did in the previous question, it would be more convenient to find a function that converts Celsius to Fahrenheit.

What we found in question #6: **Celsius = f(Fahrenheit)**

What we want in this question: **Fahrenheit = f(Celsius)**

We want to reverse the domain and range of our original function to create a new function. This is the idea of an **inverse function**, where the input becomes the output and the output becomes the input.

Before we calculate inverse functions, it's important to know whether a function even has an inverse.

Suppose we conduct a *horizontal line test*. We select any output (value of Y) and see how many inputs (X values) we get. To pass this test, each output on the graph must correspond with exactly one input. Functions that pass this horizontal line test are said to be invertible (one-to-one).

Determine which of the following functions are one-to-one (invertible):

$y = x^5 + 7$ $\qquad$ $y = |x|$ $\qquad$ $y = 4(2^x)$ $\qquad$ $y = x^6 + 2x^2 - 10$

9) The range of a function becomes the domain of an inverse function. In this way, inverse functions "undo" each other. In other words, if f(x) and g(x) are inverse functions, then...

f(g(x)) = g(f(x)) = ____________

10) Determine if the following pairs of functions are inverses by finding their compositions.

$f(x) = 8x + 5$

$g(x) = \frac{x-5}{8}$

$j(x) = x^3 + 4$

$k(x) = \sqrt[3]{x+4}$

11) Once we know a function is invertible, we can easily find the inverse by:

a) Interchanging x and y (turn the range into the domain)
b) Solving for y

We then symbolize the inverse function as $f^{-1}(x)$. Note that the "-1" does not represent an exponent.

Determine if the following functions are invertible. If they are, find the inverse function. Then, graph the original function and its inverse. What do you notice about their graphs?

$f(x) = 7x - 2$

$g(x) = \sqrt{x+5}$

$h(x) = \frac{1}{x}$

$h(x) = x^2 + 3$