MATH 171 Activity #9: Quadratic Functions	Homework: 3.5: 9, 33-36, 49-58, 59, 63 3.2: 49, 53, 54, 57 (share in class)

Situation: A Roth IRA (Individual Retirement Account) is a personal savings plan that allows individuals to contribute up to $3000 per year. Since taxes have already been paid on the contributions, an individual's money grows tax-free. Suppose you start a Roth IRA with $3000 and continue to contribute $3000 each year.

1. The amount of money at any given time in your IRA is a function of (1) the interest rate it earns and (2) time. What annual interest rate must your money earn in order to have $21,000 in five years? Go ahead and guess.

2. Right now, savings plans offer a little less than 2% annual interest. Let's create a table to see how much money we'd have in five years if we earned 2% each year.

Year	**Amount of money in the IRA**
t = 0	3000
t = 1	
t = 2	
t = 3	
t = 4	
t = 5	

3. How would you determine the amount of interest needed to get $21,000 in five years?

4. Since we don't know the interest rate needed, let's call it *x*. The following table shows how much money we'd have each year if we earned (x – 1)% interest.

Year	**Amount of money in the IRA**					
t = 0	3000					
t = 1	$3000x$	$+3000$				
t = 2	$3000x^2$	$+3000x$	$+3000$			
t = 3	$3000x^3$	$+3000x^2$	$+3000x$	$+3000$		
t = 4	$3000x^4$	$+3000x^3$	$+3000x^2$	$+3000x$	$+3000$	
t = 5	$3000x^5$	$+3000x^4$	$+3000x^3$	$+3000x^2$	$+3000x$	$+3000$

A **polynomial function** of degree *n* is defined by: $P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$

5. If our money earns (x – 1)% interest each year, the function $3000x^5 + 3000x^4 + 3000x^3 + 3000x^2 + 3000x + 3000$ models the amount of money we would have at the end of the fifth year. Graph this function in the space below. Describe the graph and estimate the interest rate needed to earn $21000 in five years.

6. As we'll see next class, we can solve polynomial equations by (a) graphing, (b) factoring, or (c) synthetic division. We'll stick with graphical methods for now. Using your calculator, graph the above function and find the interest rate needed to get $21,000 in five years. Round your answer to the nearest hundredth.

7. Let's examine the graphs of polynomial functions in detail. Before we begin, here are some of the terms we use to describe graphs. New terms are in **bold**.

Domain: All possible input values for X

Range: All possible output values (for Y)

Continuity: A function is continuous if we can draw it without lifting our pencils

Increasing: A function is increasing over an interval if it has a positive slope over that interval

Decreasing: A function is decreasing over an interval if it has a positive slope over that interval

Turning Point: A turning point is where the function changes from increasing to decreasing

Concave Up: A function is concave up if its slope is increasing (shaped like a cup)

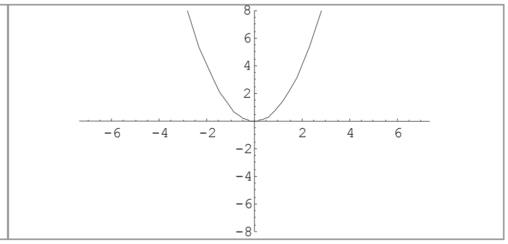
Concave Down: A function is concave down if its slope is decreasing (shaped like a frown)

Inflection Point: A point at which the function changes concavity.

Extrema: A local maximum or minimum of a function (like a vertex)

End Behavior: A comparison of the left and right ends of a graph. Do they match or do they differ?

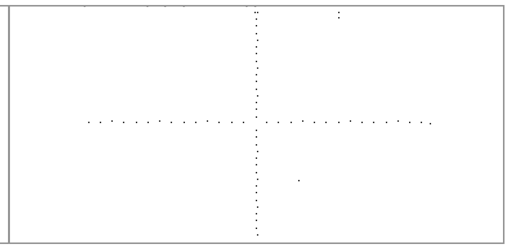
$f(x) = x^2$



Domain	Range	Continuous?	Turning Points	Extrema	Inflection Points	End Behavior

Maximum number of real zeros	Minimum number of real zeros

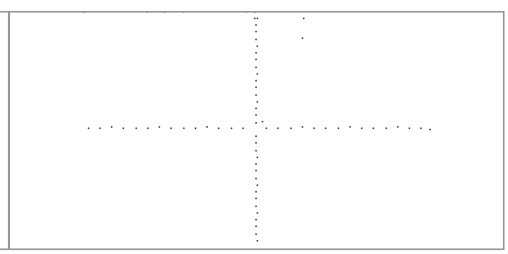
$f(x) = x^3 - 3x^2 + x - 2$



Domain	Range	Continuous?	Turning Points	Extrema	Inflection Points	End Behavior

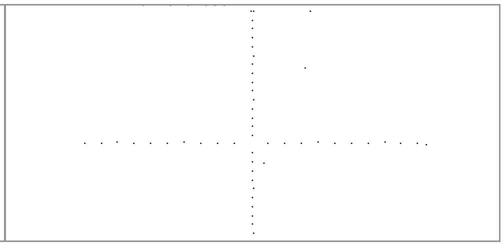
Maximum number of real zeros	Minimum number of real zeros

$f(x) = x^4 - 3x^2 + x + 0.5$



Domain	Range	Continuous?	Turning Points	Extrema	Inflection Points	End Behavior

Maximum number of zeros	Minimum number of zeros

$f(x) = x^5 + 2x^4 - x^3 + x^2 - x - 4$	

Domain	Range	Continuous?	Turning Points	Extrema	Inflection Points	End Behavior

Maximum number of real zeros	Minimum number of real zeros

8. Generalize these concepts by completing the following table:

Concept	Even-degree polynomials (degree = n)	Odd-degree polynomials (degree = n)
The domain of a polynomial		
The range of a polynomial		
Continuity of a polynomial		
End Behavior		
Maximum number of real zeros		
Minimum number of real zeros		
Maximum number of extrema		
Maximum number of inflection points		
If the leading coefficient is positive…		
If the leading coefficient is negative…		

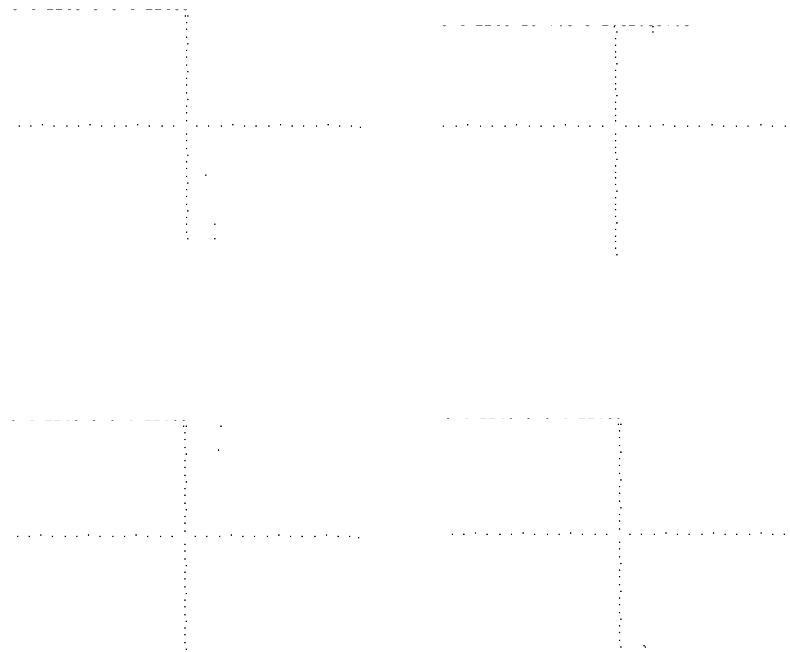
9. Match each of the following polynomial functions with its graph. Explain your reasoning.

(a) $a(x) = x^4 - x^2 + 5x - 4$

(b) $b(x) = 3x^3 - x^2 + 2x - 4$

(c) $c(x) = -x^6 + x^2 - 3x - 4$

(d) $d(x) = -x^7 + x - 4$



10. We can construct a polynomial if we're given its zeros. For example, suppose we must construct a polynomial such that its zeros are X = 0, -4, and 6. Begin by locating the zeros on the following axes. Then construct the polynomial and use your calculator to verify its zeros.

11. A polynomial can also have complex zeros. Construct a polynomial so that it has the following zeros: X = 3, 2 + i.

12. Construct a polynomial so that it has the following zeros: X = 0, -2, -3 – i.

13. How can we check our answers to the previous two questions? To check the real zeros, we can simply graph our constructed polynomial. To find the complex zeros, we need to factor the polynomial through synthetic division. Check your answer to question #11 by using synthetic division.

$$
\begin{array}{r|cccc}
 & & & \multicolumn{2}{r}{x^2-4x+5} \\ \hline
 & x^3 & -7x & +17x & -15 \\
 & x^3 & -3x^2 & & \\
(x-3) & & -4x^2 & +17x & -15 \\
 & & -4x^2 & +12x & \\
 & & & 5x & -15 \\
 & & & 5x & -15
\end{array}
$$