MATH 171 Activity #11: Rational Functions	Homework: 4.1: 9-14 all 4.2: 1-8 all, 11, 15, 21 (skip obliques) 4.3: 63, 65, 75, 87 (share in class)

Situation: Amgen, the world's largest biotechnology company, produces *Aranesp* (a treatment for anemia and kidney disease). The cost to produce *x* grams of Aranesp is modeled by: $C(x) = 4{,}750{,}000 + 5000x$

1. Explain what *4,750,000* and *5000x* represent in the function.

2. Suppose Amgen produces 100 grams of Aranesp. How high must they set the price in order to break even?

3. The price Amgen must charge to break even is modeled by: $A(x) = \dfrac{C(x)}{x} = \dfrac{4{,}750{,}000 + 5000x}{x}$. How much must they charge if they produce 10 kilograms, 100 kilograms, or 1000 kilograms of Aranesp?

4. Graph the average cost function A(x) on an appropriate window. Sketch the graph below and identify its domain and range.

5. Will Amgen ever be able to charge less than $5000 per gram of Aranesp?

A **rational function** is a ratio of two polynomials: $R(x) = \dfrac{P(x)}{Q(x)}$ where $Q(x) \neq 0$

6. Graph the function $f(x) = \dfrac{x^2 + x - 8}{x - 2}$ and identify its domain and range.

For a rational function f(x): if $f(x) \to \pm\infty$ as $x \to a$, then $x = a$ is a vertical asymptote

7. Graph the function $f(x) = \dfrac{2x + 1}{x - 3}$ and identify its domain and range.

For a rational function f(x): if $x \to \pm\infty$ as $f(x) \to a$, then $y = a$ is a horizontal asymptote

To find the asymptotes of a rational function:

1. Vertical asymptotes: Set the denominator equal to zero and solve.

2. Horizontal asymptotes:
   (a) If the degree of the numerator is less than the degree of the denominator; y=0.

   (b) If the numerator and denominator have the same degree, divide all terms by the x term of the largest degree and find the limit as $x \to \infty$

   (c) If the degree of the numerator is exactly one more than the degree of the denominator, then the rational function has an oblique asymptote.

8. Find the vertical and horizontal asymptotes for the following rational functions.

$$a(x) = \frac{x+1}{2x^2 + 5x - 3}$$

$$b(x) = \frac{6x^5 + 4x^4 + 2x - 1}{8x^5 - 3x^3 + 2}$$

$$a(x) = \frac{x^2 - 4}{x - 2}$$