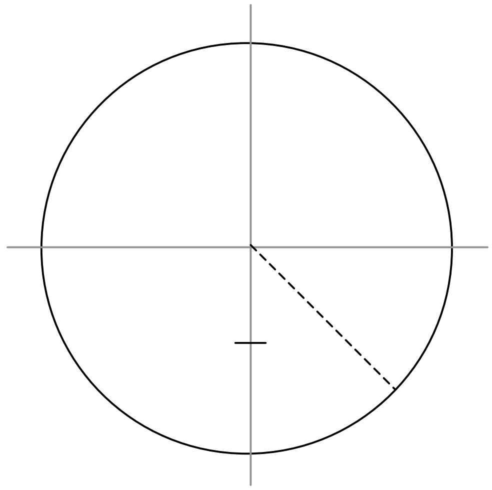
MATH 171 Activity #12: Periodic Functions	Homework: 8.1: 71 8.2: 31-45 odd

Situation: To celebrate the new millennium, British Airways announced in 1996 its plans to fund construction of the world's largest ferris wheel. The wheel was designed to measure 500 feet in diameter and to carry 1400 passengers in 60 capsules. The wheel was designed to be slow enough for people to hop on and off while it turns – it completes a single rotation once every 20 minutes.



1. Complete the following table:

t (minutes)	0	5	10	15	20	25	30	35	40
f(t) (feet)	0								0

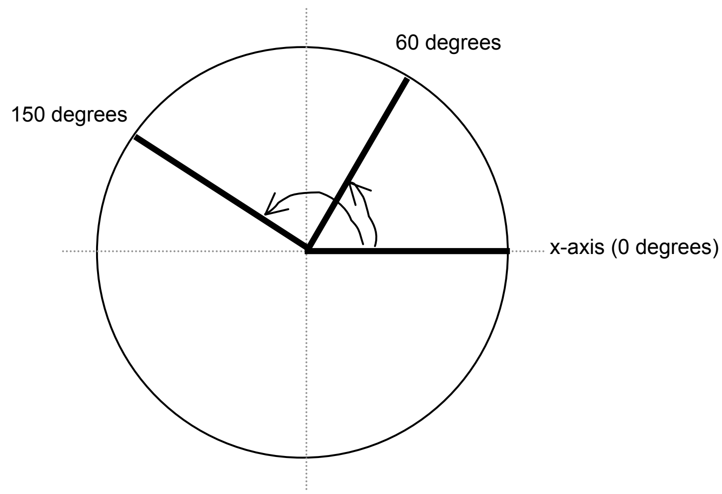
2. Plot these points on the axes. Should you connect the points with straight lines?

This graph is an example of a *periodic function*. The shortest amount of time in which the function completes one full cycle is called its *period*. The period of this ferris wheel function is 20 minutes. Notice that if the graph is shifted to the right or left by its period (20 minutes), the graph looks exactly the same.

The *amplitude* of a function is one half of the distance between its minimum and maximum values. The amplitude of the ferris wheel function is 250 feet.

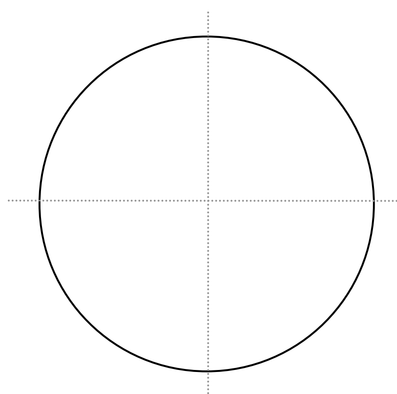
Now let's attempt to construct a formula for the ferris wheel function. To do this, we first represent positions on the ferris wheel using angles.

By convention, angles are measured counterclockwise from the x-axis. For example, the following diagram displays angles of 60 and 150 degrees.

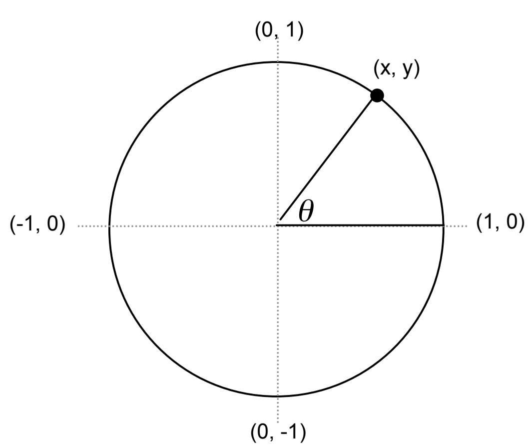


It's often useful to think of angles as rotations, since then we can make sense of angles greater than 360 degrees. We can also make sense of negative degree angles.

3. Locate points on the ferris wheel corresponding to 720 degrees, -90 degrees, and -380 degrees.



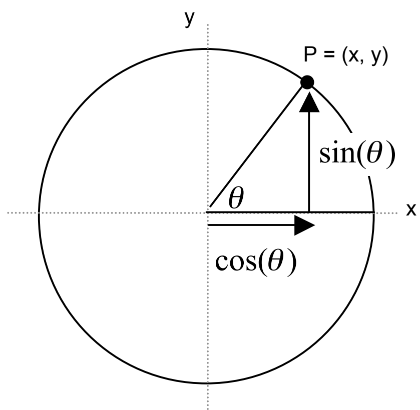
A *unit circle* is the circle centered at the origin with a radius of one unit. The distance from any point (x, y) to the origin (by the distance formula we learned the 2nd day of class), is $d = \sqrt{x^2 + y^2} = 1$



Given point P = (x, y) on the unit circle, we define the *cosine of* θ and the *sine of* θ by the formulas:

$$\cos(\theta) = x \qquad \text{and} \qquad \sin(\theta) = y$$

In other words, cos(θ) is the x-coordinate of the point on the unit circle specified by the angle θ and sin(θ) is the y-coordinate.



4. Using the above definition, evaluate the following:

(a) $\sin(0)$

(e) $\cos(0)$

(b) $\sin(90)$

(f) $\cos(90)$

(c) $\sin(180)$

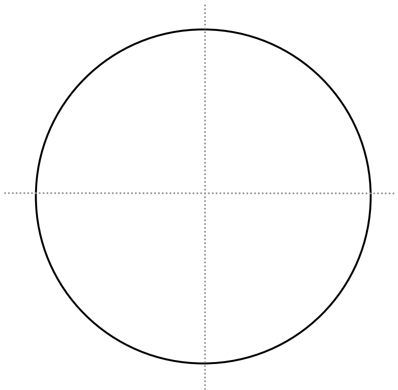
(g) $\cos(180)$

(d) $\sin(270)$

(h) $\cos(270)$

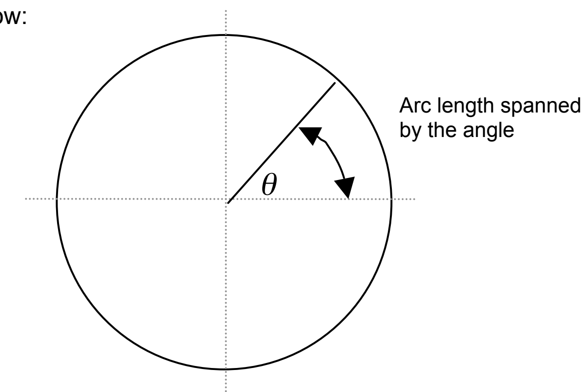
5. The point Q on the unit circle is designated by a 130 degree angle. Sketch the unit circle and the point. Then find the coordinates of point Q.

6. The ferris wheel has a radius of 250 feet. Find your height above ground as a function of the angle θ measured from the x-axis. What is your height when the angle is 60 degrees?



So far, we have measured angles in degrees. There is another way to measure an angle, which involves comparing the length of arc that the angle cuts off on a circle to the radius of the circle. This is the idea of radians, which turn out to be very helpful in calculus.

The arc length *spanned*, or cut-off, by an angle is shown below:



On the unit circle, the arc length is completely determined by the angle θ.

> An angle of **1 radian** is defined to be the angle at the center of the unit circle which spans an arc length of 1.
>
> An angle of **2 radians** is defined to be the angle at the center of the unit circle which spans an arc length of 2.
>
> An angle of **–0.6 radians** is defined to be the angle at the center of the unit circle (measured clockwise) that spans an arc length of 0.6.

7. What is the formula for the circumference of a circle with radius r? What's the circumference of a unit circle? What is the length of the arc spanned by a complete 360-degree rotation around a unit circle?

8. Now that we know 360 degrees is equivalent to 2π radians, we can convert any angle to radians using a proportion. Convert 3 radians to degrees. Then convert 3 degrees to radians.