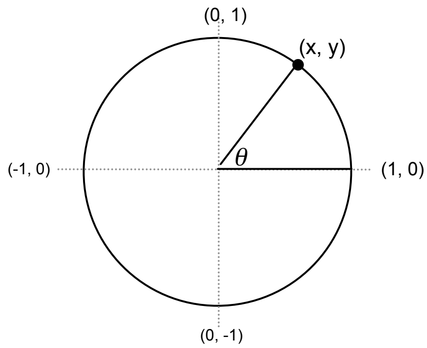
MATH 171 Activity #13: Trig Functions Homework: 8.5: 69-83 odd, 87, 89

Let's begin by reviewing trigonometric functions. Complete the table by finding the exact values of all six trigonometric functions.

Given point (x, y) on the unit circle defined by the angle θ ,

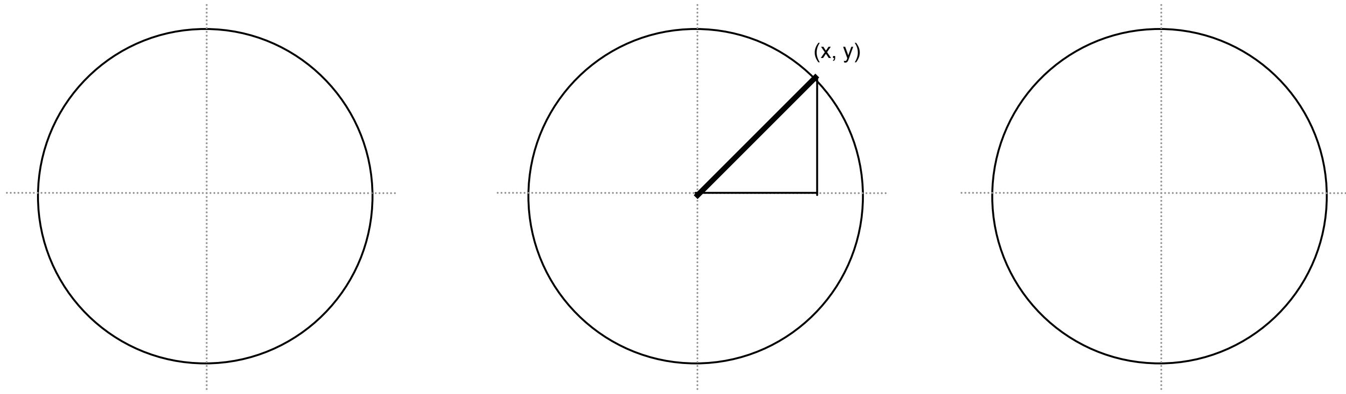


$\sin(\theta) = \frac{y}{r}$ $\cos(\theta) = \frac{x}{r}$ $\tan(\theta) = \frac{y}{x}$

$\csc(\theta) = \frac{r}{y}$ $\sec(\theta) = \frac{r}{x}$ $\cot(\theta) = \frac{x}{y}$

Notice the trigonometric functions on the bottom row are the reciprocals of the functions on the top row

Degrees	Radians	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
0	0	0	1	0	--	1	--
30							
45							
60							
90							
120							
135							
150							
180	π	**0**	**-1**	**0**	**--**	**-1**	**--**
210							
225							
240							
270							
300							
315							
330							
360	2π	**0**	**1**	**0**	**--**	**1**	**--**



To complete the table, you should:

1. Convert 90-degrees to radians and use this information to convert 270-degrees to radians
2. Convert 45-degrees to radians and use this information to convert 135-, 225-, and 315-degrees to radians
3. Convert 30-degrees to radians and use this information to complete the first column

4. Use the first blank unit circle to plot points corresponding to 90, 180, 270, and 360 degrees.
5. Find the values of the trigonometric functions for these angles.

Finding the exact values for the 45-degree angle is a bit tougher. A 45-degree angle has been drawn on the second unit circle. We know three things about the point (x, y):

1. We know the distance from the point (x,y) to the origin is 1, because this is a unit circle.

2. The distance formula tells us that $x^2 + y^2 = 1$ for the unit circle

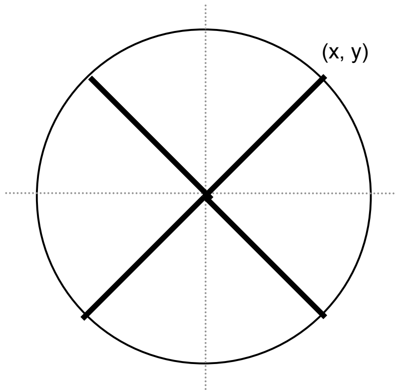
3. We know the line drawn from the origin to the point (x, y) is defined by y = x

We can thus solve the following system of linear equations:

$$\begin{aligned} x^2 + y^2 &= 1 \\ y &= x \end{aligned}$$

Each group will be responsible for completing one column of the table and graphing their trigonometric function. Specify the domain and range of your function.

Notice the $\tan(\theta)$ function is simply the slope of the line passing from the origin through the point (x, y). Over what intervals is the slope of the line positive? Over what intervals is it negative?



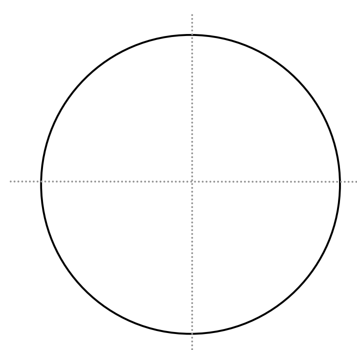
Relationships among trigonometric functions:

$$\tan(\theta) = \frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r} = \frac{y}{r}\left(\frac{r}{x}\right) = \frac{y}{x}$$

$$\csc(\theta) = \frac{1}{\sin\theta} = \frac{1}{y/r} = 1\left(\frac{r}{y}\right) = \frac{r}{y}$$

$$\sec(\theta) = \frac{1}{\cos\theta} = \frac{1}{x/r} = 1\left(\frac{r}{x}\right) = \frac{r}{x}$$

$$\cot(\theta) = \frac{\cos\theta}{\sin\theta} = \frac{x/r}{y/r} = \frac{x}{r}\left(\frac{r}{y}\right) = \frac{x}{y} = \frac{1}{\tan\theta}$$



We have seen that the coordinates of any point (x, y) on the unit circle are: $(\cos\theta, \sin\theta)$

We also know that the distance from the point (x, y) to the origin is given by: $\sqrt{x^2 + y^2} = 1$ or $x^2 + y^2 = 1$

Therefore, $\sin^2\theta + \cos^2\theta = 1$.

This is the fundamental trigonometric identity you should memorize for calculus.

We can use this fundamental identity to derive other identities:

$$\sin^2\theta + \cos^2\theta = 1$$

Divide each term by $\sin^2\theta$:

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1}{\sin\theta}\right)^2$$

$$1 + (\cot\theta)^2 = (\csc\theta)^2$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

Divide each term by $\cos^2\theta$:

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$$

$$(\tan\theta)^2 + 1 = (\sec\theta)^2$$

$$\tan^2\theta + 1 = \sec^2\theta$$