MATH 171 Activity #3: Linear Systems	Homework: 7.1: 7, 13, 19, 31, 39, 94 7.2: 5, 13, 21

A *linear system* is a set of two or more linear equations. The *solution* to a linear system tells us where the lines intersect.

Linear systems come in three flavors: (1) inconsistent, (2) dependent, and (3) consistent and independent

1. An *inconsistent* linear system has no solution (the lines never intersect). Sketch and give an example of an inconsistent system.

2. A *dependent* linear system has infinitely many solutions. Sketch and give an example of a dependent system.

3. A system that is independent and consistent has exactly one solution. Sketch and give an example of this type of linear system.

We can solve linear systems (find intersection points) using the following methods:

1. Substitution Method
2. Elimination Method
3. Echelon Method
4. Cramer's Rule
5. Matrix Inverses

In this class, we'll concentrate on the substitution and elimination methods.

4. Let's go back to the example where you're selling coffee out of my office. Recall that your costs (as a function of the number of cups of coffee you make) are modeled by:

$$C(x) = 20 + 0.25x$$

Suppose you decide to sell each cup of coffee for $1.50. Write out the formula expressing your revenues as a function of the number of cups of coffee you sell.

How many cups must you sell in order to break-even? Answer this question by first graphing both linear functions. Use the INTERSECT feature of your graphing calculator to find the solution.

5. Solve the following linear systems via the substitution method:

$$\begin{cases} 3x - 4y = 9 \\ 2x - 8y = -10 \end{cases}$$

$$\begin{cases} 8x + 3y = 2 \\ 5x = 17 + 6y \end{cases}$$

6. Another method (that students typically prefer) used to solve linear systems is the elimination method. The goal of this method is to get rid of one of the variables.

   With the elimination method, we first multiply the equations on both sides by suitable numbers so that when they are added together, one variable is eliminated.

   Let's go through an example of the elimination method:

$$\begin{cases} 3x - 4y = 1 \\ 2x + 3y = 12 \end{cases}$$

1. Here is our linear system. Arbitrarily, I decide that I want to eliminate the x variable. To do this, I need the coefficients of X in both equations to be opposite. I notice that I could change the coefficients of 3 and 2 into 6.

$$\begin{cases} 6x - 8y = 2 \\ 6x + 9y = 36 \end{cases}$$

2. I multiply the terms in the top equation by 2 and the terms in the bottom equation by 3. Notice that the coefficients are now the same. I need the coefficients to be opposite.

$$\begin{cases} -6x + 8y = -2 \\ 6x + 9y = 36 \end{cases}$$

3. This can be done by multiplying the top equation by -1.

$$17y = 34$$

4. I then add the equations together. The x term is eliminated.

$$y = 2$$

5. Dividing both sides by 17, I find the solution for y. Now I need to find the value of X.

$$3x - 4(2) = 1$$
$$3x - 8 = 1$$
$$3x = 9$$
$$x = 3$$

6. To do this, I can substitute y = 2 into one of the original equations.

   The solution to this linear system is: X = 3, Y = 2

7. Solve the following system of linear equations using the elimination method: $\begin{cases} 5x + 7y = 6 \\ 10x - 3y = 46 \end{cases}$

8. Solve the following system of linear equations using the elimination method: $\begin{cases} 3x - 2y = 4 \\ -6x + 4y = 7 \end{cases}$

9. Solve the following system of linear equations using the elimination method: $\begin{cases} 8x - 2y = -4 \\ -4x + y = 2 \end{cases}$

10. The Title IX legislation prohibits sexual discrimination in sports programs. In 1997, the national average spent on one female and one male athlete was \$6050 for Division I-A schools. However, average expenditures for males exceeded those for female athletes by \$3900. Determine how much was spent per male and female athlete.

11. We can also use the elimination method to solve a system of 3 or more linear equations. To do this, we can take the following steps:

    1. Pick a variable (x, y, or z) to eliminate.
    2. Choose two of the linear equations and use the elimination method to eliminate your chosen variable
    3. Choose a different pair of your linear equations and use the elimination method to eliminate the same variable
    4. You now have two linear equations with two unknown variables. You can solve this system easily.

    Solve the following system of linear equations: $\begin{cases} 4x - 3y + z = 9 \\ 3x + 2y - 2z = 4 \\ x - y + 3z = 5 \end{cases}$

Solve homework problems.