

We've studied linear functions in-depth. Now we'll use what we've learned to investigate piecewise functions and absolute functions.

A *piecewise-defined* function is "defined by different rules over different subsets of its domain." In plain English, a piecewise function is a bunch of functions put together. Consider the *absolute value* function (a simple example of a piecewise-defined function):

1. Complete the following table. Graph  $f(x)$  and  $g(x)$  over the interval  $(-6, 6)$ .

$x$	$f(x) = x - 3$	$g(x) =  x - 3 $
-6		
-4		
-2		
0		
2		
4		
6		

2. Explain the difference between the two functions. Identify the domain and range of each function.

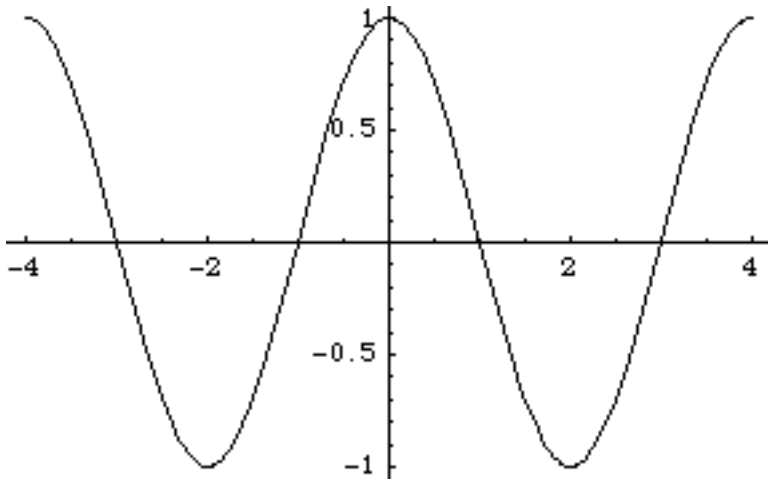
The absolute value function,  $g(x) = |x|$ , is never negative. If we input a positive value for  $x$ , the function yields a positive output. If we input a negative value for  $x$ , the function still yields a positive output. Therefore, we can write the absolute value function as a piecewise-defined function.

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

We can use absolute values in two ways to create new functions from old. First, we can take the absolute value of the output of a function. Second, we can take the absolute value of the input of a function.

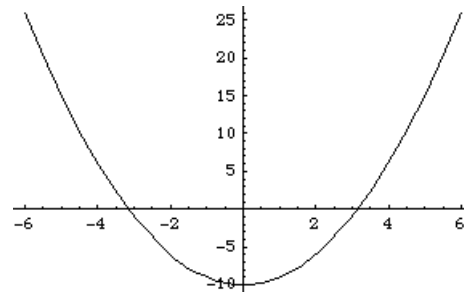
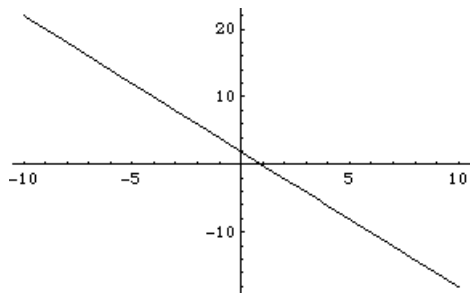
In other words, given a function  $f(x)$ , we can create the following new functions:  $y = |f(x)|$  and  $y = f(|x|)$

3. The graph of the function  $f(x) = 10 \cos\left(\frac{\pi x}{2}\right)$  is displayed below. Your goal is to graph and clearly label the following:  
 $g(x) = |f(x)|$  and  $h(x) = f(|x|)$



Explain how  $g(x)$  and  $h(x)$  differ from the original function. Identify the domain and range of each function.

4. For each of the following functions, sketch the graphs of:  $g(x) = |f(x)|$  and  $h(x) = f(|x|)$



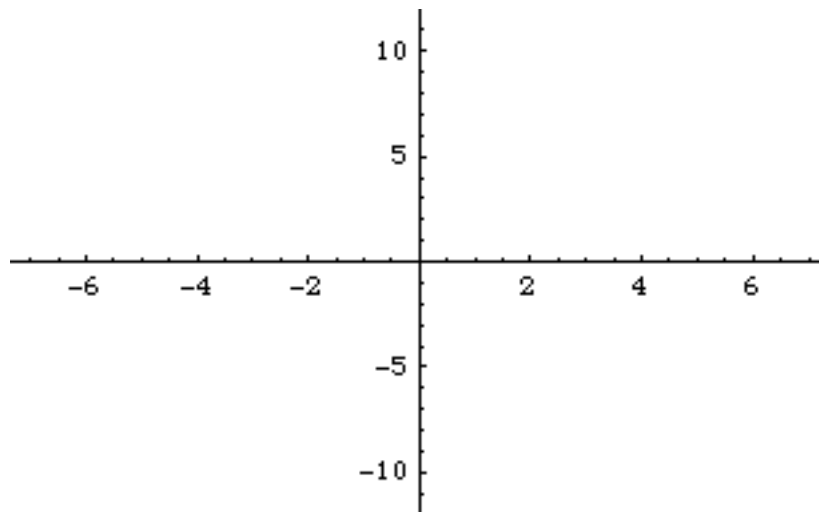
5. Just as we solved linear equations analytically and graphically, we can solve absolute value equations and inequalities. Solve the following analytically and/or graphically

$$|x - 4| = |7x + 12|$$

$$|x - 4| > |7x + 12|, \text{ solve graphically}$$

6. As was stated earlier, a piecewise-defined function is defined by different rules over different subsets of its domain. Here's an example of a piecewise function:

$$f(x) = \begin{cases} 3x + 1 & , x \leq 0 \\ 1 & , 0 < x < 4 \\ (x - 3)^2 & , x \geq 4 \end{cases} \quad \text{Graph this function on the following axes.}$$



To graph this function on your calculator, you need to enter the following:

$$Y_1 = (3x + 1) * (x \leq 0)$$

$$Y_2 = (1) * (x > 0)(x < 4)$$

$$Y_3 = ((x - 3)^2) * (x \geq 4)$$

The inequality symbols are listed under **TEST** (above the **MATH** key)

Situation: In a triathlon, an athlete's distance is a function of time. Suppose a triathlon consists of the following legs:

1. You swim for 2.4 miles at 2 mph
2. You ride a bicycle at 20 mph for 112 miles
3. You run a 26.2 mile marathon at a speed of 9 mph

Your goal is to graph your distance as a function of time and write out the piecewise function

Extra-credit Question (worth quite a few points):

Three telecommunications companies offer the following wireless service plans.

Company A: \$17.95 monthly fee plus 73 cents per minute

Company B: \$27.95 monthly fee includes 15 free minutes. Each additional minute is 53 cents

Company C: \$49.95 monthly fee includes 60 free minutes. Each additional minute is 35 cents

Company D: \$75.95 monthly fee includes 150 free minutes. Each additional minute is 35 cents.

Graph the monthly bill as a function of the minutes used. Then write the piecewise function for the cheapest service plan from 0 – 180 minutes.