

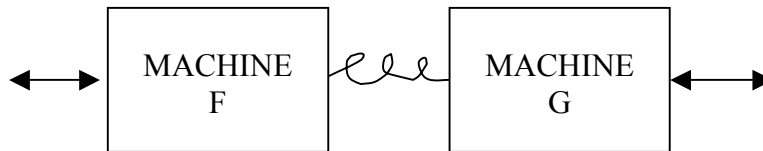
1. On your way to math class, you stumble across a magic lamp. After rubbing it, a genie appears and grants you 3 wishes. Since you're a typical college student, you wish for an "automatic money machine."

The genie grants your wish and an ATM-like machine appears. The genie tells you the machine will automatically add \$50 to whatever amount of money you insert into it. We'll call this Machine F.

You realize it will take too long to get rich from this machine, so you wish for a second machine that will automatically double any amount of money you insert. We'll call this Machine G.

While trying the machines out, you say, "Boy, I wish I had an adapter to hook these machines together." A few seconds later, the machines are magically connected.

These machines can be visualized as:



You can put money into either side of the machines.

2. Write out formulas for Machine F & G, showing what happens when you insert \$X into each machine separately.
3. If you put money in on the right side, you are first finding $f(x)$ and then evaluating $g(x)$. This is written as $g(f(x))$.

Find the value of $g(f(5))$, $g(f(10))$, and $g(f(x))$.

4. If you put money in on the left side, you are first finding $g(x)$ and then evaluating $f(x)$. This is written as $f(g(x))$.

Find the value of $f(g(5))$, $f(g(10))$, and $f(g(x))$.

5. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$. Perform the following operations

$f(g(x))$ _____

$g(f(x))$ _____

6. Complete the following table:

x	0	1	2
m(x)	1	9	_____

h(x)	2	0	1
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m(h(x))	5	_____	_____
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7. How can we tell if a graph represents a function? Why does this work?

8. Suppose we conduct a *horizontal line test*. If a graph that passes this test, it means that each value of y (output) corresponds with exactly one value of x (input). Functions that pass the horizontal line test are said to be invertible (or one-to-one).

Determine which of the following functions are one-to-one.

$$y = x^5 + 7$$

$$y = |x|$$

$$y = 4(2^x)$$

$$y = x^6 + 2x^2 - 10$$

9. The range of a function becomes the domain of an inverse function. In this way, inverse functions “undo” each other. In other words, if $f(x)$ and $g(x)$ are inverse functions, then $f(g(x)) = g(f(x)) = x$.

Determine if the following pairs of functions are inverses by finding their compositions.

$$f(x) = 8x + 5$$

$$g(x) = \frac{x - 5}{8}$$

$$j(x) = x^3 + 4$$

$$k(x) = \sqrt[3]{x + 4}$$

10. Once we're sure a function is invertible, finding an inverse function is a simple 2-step process:

- (1) Interchange y and x . (Turn the range into the domain)
- (2) Solve for y to find the inverse function.

We symbolize the inverse function as $f^{-1}(x)$.

11. Find inverse functions for the following functions. Then graph both the original function and its inverse. What do you notice about the graph of a function and its inverse?

$$f(x) = 7x - 2$$

$$g(x) = \sqrt{x + 5}$$

$$h(x) = \frac{1}{x}$$

$$h(x) = x^2 + 3$$