MATH 171 Activity #7: Exponential/Logarithmic Functions	Homework: 5.2: 13 – 18 all, 35 5.3: 19, 81 5.4: 9, 23 – 30 all

1. Suppose we live in a magical land where bunnies never get old or die. When we first arrive, we count 1,000 rabbits. One month later (when we get bored enough to start counting rabbits again), we count 1,100 bunnies. With nothing better to do, we decide to create a model of the population growth of these rabbits.

   First, let's assume the rabbit population grows at a linear rate. Complete the table and write out the linear model.

Month	Number of rabbits	Change from previous month
0	1000	--
1		
2		
10		
60		
61		
M		

2. Explain why this is not a realistic model for the population growth of rabbits. How would you improve the model?

3. Since our linear model isn't realistic, we need to come up with a new model. By what percentage did the rabbit population increase in the first month? Let's assume the rabbits increase by that percentage each and every month. Complete the table.

Month	Rabbits	% change from previous month	Formula
0	1000	--	
1			
2			
3			
4			
61			
m			

4. Write out the general formula you found in the previous question and graph it below. Identify the domain and range of this function.

An **Exponential Function** is defined by: $f(t) = a(1+r)^t$ where $a > 0$

5. Explain what the *a* represents in the formula for an exponential function. Also explain what the *r* represents.

6. Before we go through any more examples of exponential functions, we need to review some basic facts about percents. Answer the following questions:

   a. What is 12% of 475?

   b. We invest $1000 in 1998 and receive $3000 in 2000. What is the percentage change?

   c. We invest $1000 in 2001 and receive $380 in 2003. What is the percentage change?

   d. If we multiply a number by 1.15, it increases by ______%. If we multiply a number by 0.72, it decreases by ______%.

7. A cell divides every minute. How many cells will there be in one hour? Use an exponential model.

8. After graduating, you get a job at a starting salary of $30,000. You are promised a 4% raise each year. In your lifetime, will you ever reach a $100,000 salary?

9. Graph the following two functions and describe what happens as we increase the value of *a*:

$$f(x) = 3(1.3)^x$$

$$g(x) = 5(1.3)^x$$

10. A cup of coffee contains 100 mg of caffeine. Each hour, your body metabolizes 16% of the caffeine. Suppose you drank 2 cups of coffee at 8am. How much caffeine remains in your body at 9am? How about at noon? 8pm? When will all the caffeine be eliminated from your system? Graph this function.

11. Graph the following three functions and describe what happens as we change *(1+r)*:

$$f(x) = 200(1.2^x)$$
$$g(x) = 200(1.0^x)$$
$$h(x) = 200(0.8^x)$$

12. The following table displays historical returns for stocks, bonds, and money market accounts from 1925 to 2003 (source: American Century Services Corporation using data presented by Ibbotson Associates, Inc.):

Stocks	Bonds	Money Market
10.3%	5.5%	3.8%

Let's assume these averages remain true for the next 40 years. Suppose you invest $3000 at age 20 into one of these asset classes. Write out the exponential models for each asset class. Then determine how much money you would have at age 60 (assuming you do not add any more money and the money grows tax-free).

13. You invest $100 in an S&P 500 index fund that grows 10% each year. Write out the formula that models the growth of your money over time. How much will you have in 8 years?

14. How long will it take to reach $2000? Is this an exact answer or an approximate?

To get an exact answer to this problem, we must use logarithms.

A **logarithm** is the inverse of the exponential function

Huh? What's that mean?

Recall: The functions f(x) and g(x) are inverses if $f(g(x)) = g(f(x)) = x$.

15. Write out the general formula for an exponential function and go through the process used to find its inverse.

Assuming x > 0, log(x) is simply the exponent of 10 that yields x. In other words, if $10^y = x$, $y = \log(x)$

16. Use your calculator to verify the answers to the following logarithms:

(a) $\log(100) =$ exponent of 10 that yields 100 or $10^x = 100$. We know $10^2 = 100$, so $\log(100) = 2$

(b) $\log(1000) =$

(c) $\log(10) =$

(d) $\log(1) =$

(e) $\log(0.1) = \log(\frac{1}{10}) =$

(f) $\log(0.01) =$

(g) $\log(-10) =$

17. Estimate the following:

(a) $\log(250) =$

(b) $\log(5000) =$

18. If you're given $10^{2.8} = 630.957$, then $\log(630.957) =$ ___________?

The output of a logarithm is simply an exponent.

$\log_6 36 \Rightarrow 6^x = 36 \Rightarrow x = 2$

$\log_2 1 \Rightarrow 2^x = 1 \Rightarrow x = 0$

$\log_{100} 10000 \Rightarrow 100^x = 10000 \Rightarrow x = 2$

$\log_3 27 \Rightarrow 3^x = 27 \Rightarrow x = 3$

Generalized: $\log_a(a^n) = n$. A logarithm is simply an exponent.

19. Solve for x: $480(10^{0.06x}) = 1320$

Properties of Logarithms:

$\log(x) = y$ means $x = 10^y$

$\log(1) = 0$ since $1 = 10^0$

$\log(10^x) = x$ and $10^{\log(x)} = x$ (because they are inverses)

$\log(ab) = \log(a) + \log(b)$

$\log(\frac{a}{b}) = \log(a) - \log(b)$

$\log(b^t) = t\log(b)$

20. Solve the following: $10^{.5x} = 10{,}000$

$\log(10^{banana}) =$ __________

21. Solve the following using the properties of logarithms:

(a) $\log(3x+2)+1=0$

(b) $4(1.171^x)=7(1.088^x)$

(c) $\ln(e^{2x})=$

(d) $121e^{-.112t}=88$

Solving exponential or logarithmic equations:

$a^{f(x)}=b$	Take the logarithm of both sides
$\log(f(x))=b$	Raise 10 to the power of each side
$\log(f(x))=\log(g(x))$	Simply set f(x) = g(x)