MATH 171 Activity #9: Quadratic Functions	Homework:	3.1:	31, 45
		3.2:	7, 13, 17, 27, 29, 45, 49,53, 55
		3.3:	17, 21, 23, 71, 73
		3.4:	3, 5, 7, 9, 11 (share in class)

Situation: Your part-time job isn't enough to pay your tuition, so you decide to start your own business. After brainstorming ideas, you conclude the best way to make money is by raising alpacas. You go to http://www.alpacanet.com/ and successfully bid on a six alpacas, three males and three females. You also buy 1200 yards of fencing and a plot of land to raise your alpacas.

Since you can't handle any additional alpacas right now, you decide to use the fence to create two adjacent areas for alpacas (one for the males; one for the females). What is the largest possible area you can get from your 1200 yards of fencing?

Additional Info: You do not need equal areas for the males and females.
You only have the skills to create a rectangular area.

1. Draw and label a picture of this situation.

2. Use the given information to write out formulas of interest.

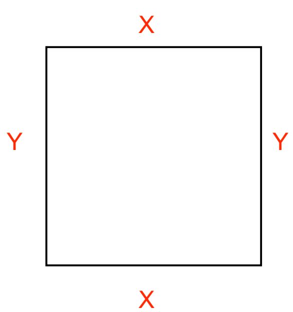
3. We have two equations that we cannot solve separately. We can use a substitution method to solve them, however. Use substitution to find the formula for the area of interest.

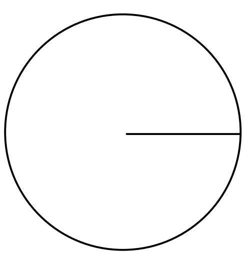
Quadratic Functions are defined by: $f(x) = ax^2 + bx + c$

4. Graph the (quadratic) area function. Using your calculator, find the maximum area that can be created from 1200 feet of fence. What are the domain and range of this function? Over what intervals is it increasing/decreasing? Is it concave up or concave down?

5. Write out the answer to the original question.

6. Bonus question: Suppose we needed just one large area (we don't need to separate males and females). What is the maximum area you could enclose with 1200 yards of fence? Suppose you could make one large circle from the fencing, what area would be enclosed by the circle?





Vertex Form of a Quadratic: $f(x) = a(x-h)^2 + k$

where: Vertex is located at the point (h, k)
The axis of symmetry is located along $x = h$
Concave up if $a > 0$; concave down if $a < 0$

7. Identify the location of the vertex and graph the following quadratic functions:

(a) $f(x) = -2(x+4)^2 - 7$

(b) $g(x) = 3(x-2)^2$

(c) $y = x^2 - 6x + 8$

8. To get the previous formula into vertex form, we must *complete the square*.

Completing the Square: To transform $f(x) = ax^2 + bx + c$ into $f(x) = a(x-h)^2 + k$:

1. Divide each side of the equation by *a*
2. Calculate $\frac{b^2}{4a^2}$ (where *a* is now equal to one).
3. Add that value after $\frac{b}{a}x$ and subtract it after $\frac{c}{a}$
4. Factor $a\left[\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a^2}\right]$

9. Complete the square for the following quadratic functions. Then locate the vertex and graph.

(a) $x^2 - 6x + 8$

(a) $-4x^2 - 12x - 8$

10. In this class, we'll use the MAXIMUM and MINIMUM functions of our calculators to find the vertices of quadratic functions. Write out a quadratic function that has a minimum and find the coordinates of the minimum. Then write a quadratic function with a local maximum and find the coordinates of that maximum.

11. A formula you will run across in calculus (you will most likely derive it) gives the height of a projectile as a function of time.

Projectile: Ignoring air resistance, the height s of an object projected upward from an initial height of s_0 feet with initial velocity v_0 feet per second is:

$s(t) = -16t^2 + v_0 t + s_0$ where *t* = time in seconds

Situation: Roger Clemens throws a baseball directly upward from an initial height of 6 feet with an initial velocity of 80 feet per second.

12. Answer the following questions:

(a) Give the function that describes the height of the ball as a function of time
(b) Graph the function on an appropriate window
(c) Determine the domain and range of the function
(d) Determine the height of the ball after 4 seconds
(e) At what time does the ball reach its maximum height?
(f) How high is the ball at its maximum?
(g) How many seconds does it take for the ball to hit the ground?

We're going to take a break from applications and story problems to learn how to solve quadratic equations and inequalities.

13. Graph $f(x) = 2x^2 + 4x - 16$ and find it's minimum.   What is the slope of a tangent line at that point?

14. Recall that when we *solve* a function, we're interested in finding the input values that output a value of zero. Using the ZERO or ROOT function of your calculator, solve: $2x^2 + 4x - 16 = 0$

Solving Quadratic Equations:

1. Graph the function and use the ZERO/ROOT function on your calculator
   Pros: Easy, works for real roots
   Cons: No complex roots, window might be deceiving, answers rounded

2. Factor the quadratic function and use the zero product property
   Pros: Works for many simple quadratics
   Cons: Not all quadratics can be factored

3. Quadratic Formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   Pros: Always works, finds complex roots
   Cons: Can make arithmetic mistakes

When you solve quadratic equations, you should always use two methods!

15. Solve the following quadratic equation graphically: $\frac{(x-50)^2}{20} + \pi(x-50) - 5$

16. Solve the following quadratic equation analytically: $2x^2 + 4x - 16 = 0$

17. Solve the following quadratic equation analytically and check graphically: $x^2 - 6x + 9 = 0$

18. Solve the following quadratic equation analytically and check graphically: $x^2 + 2 = 7$. Use the intersection of graphs method.

19. Instead of factoring, we can use the quadratic formula. Here is how the quadratic formula is derived. Explain what is happening at each step:

$$ax^2 + bx + c = 0$$

$$a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = 0$$

$$a\left[\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + \left(\frac{c}{a} - \frac{b^2}{4a^2}\right)\right] = 0$$

$$\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + \left(\frac{c}{a} - \frac{b^2}{4a^2}\right) = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$x + \frac{b}{2a} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \frac{b^2 - 4ac}{4a^2} \qquad x = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} - \frac{b}{2a} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

20. Use the quadratic formula to solve: $-4x^2 - 12x - 8 = 0$

21. What is the highest number of solutions we'll find to a quadratic equation? What's the fewest number of roots? Sketch a graph of each quadratic equation.

22. Use the quadratic formula to solve: $x^2 - 4x + 2 = 0$

23. Use the quadratic formula to solve: $2x^2 - x + 4 = 0$

24. While the previous quadratic equation does not have any real roots, it does have complex roots. To find these complex roots, we need to know about the imaginary number, *I*:

Imaginary Number *I* :	$i = \sqrt{-1}$	**or**	$i^2 = -1$

Solve the following:

(a) $(2 - 3i)(3 + 4i)$

(b) $\dfrac{3 + 2i}{5 - i}$

25. Use the quadratic formula to solve: $2x^2 - x + 4 = 0$