Activity #17b: Central Limit Theorem #2

1) Explain the Central Limit Theorem in your own words.

Importance of the CLT: You can standardize and use normal distribution tables to calculate probabilities for the sample mean.

2) Recall the *Long-Winded Professor* activity. The distribution of time spent lecturing after class should have ended was normal with $\mu = 5$ minutes and $\sigma = 1.804$ minutes). On any given day, what is the probability that the professor will lecture less than 5.5 minutes overtime?

3) What is the probability that the professor will lecture more than 3 minutes overtime?

4) Now suppose you observe the professor for five days and record his overtime each day. We will assume the amount of overtime the professor lectures is independent from day to day. What is the probability that the average of these five times is less than 5.5 minutes?

5) Compare your answers to #4 and #2. Which is larger? Should one be larger than another?

6) Suppose you now observe the professor and record his overtime for each of 40 days. What is the probability that he will average less than 5.5 minutes overtime for those 40 days? How does it compare to your previous answers?

Situation: Potato Chip Factory

7) You work for a potato chip factory. Suppose that the weights of bags of potato chips coming off an assembly line are normally distributed with $\mu = 12$ oz. and $\sigma = 0.4$ oz. What is the probability that one randomly selected bag weighs less than 11.9 ounces? Hint: First draw and label a sketch of the probability distribution and shade the region representing this probability

8) If you take a random sample of ten bags ($n = 10$) from the assembly line, what is the probability that the average weight of those ten bags will be less than 11.9 ounces? Do you believe it will be bigger, smaller, or about the same as your previous answer? Once again, sketch the probability distribution (with the horizontal axis labeled) and shade the region representing this probability. Does this probability indicate that a sample mean as small as 11.9 ounces would be surprising if the population mean was really 12 ounces?

9) Suppose you took 100 bags off the assembly line. What is the probability that the mean weight of those bags would be less than 11.9 ounces? Would this result surprise you?

10) Suppose you took 1000 bags off the assembly line. What is the probability that the mean weight of those bags would be less than 11.9 ounces?

11) What is the smallest sample size for which the probability of the sample mean being less than 11.9 ounces is less than 0.1? Hint: Your first step should be to find the first percentile of the standard normal distribution (finding k such that $P(Z < k) = 0.1$).

12) If you were told that a consumer group had weighed randomly selected bags and found a sample mean of 11.9 ounces, would you doubt the claim that the true mean weight of all the potato chip bags is 12 ounces? On what unspecified information does your answer depend? Explain.

13) Suppose you discovered that the weights of potato chip bags were not normally distributed – they had a heavily skewed distribution. Would your probability calculations change?

14) What is the probability that the sample mean weight from a sample of 10 randomly selected bags would be between 11.75 and 12.25 ounces?

15) What is the probability that the sample mean weight from a sample of 100 randomly selected bags would be between 11.92 and 12.08 ounces?

16) Find a value *k* such that the probability of the sample mean weight of 1000 randomly selected bags being between 12-*k* and 12+*k* is roughly 0.95.
In other words, between what two $\overline{X}$ values do the middle 95% of the $\overline{X}$ values fall?

17) What is the smallest sample size for which the probability is 0.95 that the sample mean falls within (-.05 and +0.5) of 12 ounces (i.e., between 11.95 and 12.05 ounces)?

Take-Home Assignment

1) Suppose a communications company sells aircraft communication units to civilian markets. Each month's sales depend on market conditions that cannot be predicted exactly, but the company executives predict their sales through the following probability estimates

x = # of units sold	25	40	65
p(x)	0.4	0.5	0.1

What is the expected number of units sold in one month?

Determine the variance, σ^2, of the number of units sold per month.

2) Suppose we wanted to examine the average number of units sold per month, say $\overline{X}$, for 3 years (n=36 months). Based on the CLT (and assuming the number of units sold from month to month is independent), what can you say about the sampling distribution of $\overline{X}$? Draw a sketch of this sampling distribution. Make sure the horizontal axis is labeled and scaled

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3) Approximate the probability that the average number of units sold per month in 36 months is 40 or higher. On the graph you just sketched, shade in the area corresponding to this probability.

4) Would this probability increase, decrease, or stay the same if the number of months were to increase? Explain without calculating anything.

5) Approximate the probability that the mean number of units sold in 36 months is between 35 and 40. Sketch a graph of the distribution and shade in the area corresponding to the probability of interest.