

MATH 300 Unit #2 Practice Exam

Use the following information to answer questions 1-4:

Miguel Cabrera, first baseman for the Detroit Tigers, hit .344 this season. In other words, he had a 34.4% chance of getting a hit in a randomly selected at-bat. He also hit 48 doubles in 161 games (0.298 doubles per game).

1. What is the expected number of games Cabrera will play before he hits 2 doubles? How would we calculate the median number of games needed to get 2 doubles?
2. What is the probability that Cabrera will get at least 6 hits in 15 at-bats? Is this the same as the probability that Cabrera will get more than 6 hits in 15 at-bats? Explain.
3. Suppose the Detroit Tigers play 6 games against the Texas Rangers. What's the probability that Cabrera will get no doubles in those 6 games? Note that you do not know how many at-bats Cabrera will get each game.
4. What's the probability that Cabrera will need at least 2 at-bats before getting a hit?
5. Over your lifetime, you've noticed that Cubs fans complain about losing every 15 minutes (on average). Suppose the student sitting next to you is a Cubs fan. What's the probability that student will begin to complain within 10 minutes? Is this the same as asking for the probability of waiting *one minute or less* to hear that student complain? Explain.
6. Suppose that in a class of 20 students, 15 are baseball fans. If I randomly select 5 students from this class, what's the probability that at least 4 of them are baseball fans? How do I know whether to use a binomial, hypergeometric, or negative binomial distribution to answer this question?

Use the following information to answer questions 7-10:

Depending on what foods you eat, the amount of potassium in your diet varies from day-to-day. Suppose you measure your blood potassium level every day for 10 years. Based on this data, you figure the average amount of potassium in your blood is 3.4 units (with a standard deviation of 0.2 units). Over the last 10 days, you measured the following blood potassium levels: 2.7 3.1 3.2 3.2 3.3 3.5 4.1 4.5 5.2 6.4*

7. Calculate the mean, median, mode, 25th/75th percentiles, and standard deviation of these 10 measurements. How would you calculate a trimmed or Winsorized mean? If these 10 measurements represent the overall distribution of your blood potassium levels, would you guess this distribution is symmetric or skewed? Sketch a boxplot.
8. Let's assume blood potassium levels for an average person follow a normal distribution (with a mean of 3.4 and a standard deviation of 0.2). What's the probability The mean of your last 10 measurements is 3.92. Is this an unusually high mean over a 10-day period? What's the probability of observing a 10-day average of 3.92 or higher? If we asked the same question using a 100-day average, would the probability increase or decrease? Why?
9. Looking over your last 10 measurements, you find a typo. The value of 4.1 should actually be 3.1. How will fixing this error impact the mean, median, and standard deviation?
10. Suppose you measured your blood potassium levels in *milliEquivalents per liter* (mEq/L). A Google search informs me that 1 mEq is equivalent to 39.1 milligrams of potassium. If we convert all our measurements to milligrams, what would happen to the mean, median, and standard deviation?

Use the following information to answer questions 11-12:

The ACT is designed to yield scores that follow a normal distribution. In 2010, the average ACT score was 21.0 with a standard deviation of 5.2.

11. Students with ACT scores of 21 or less will be placed into MATH 099 next year. If the 550 students entering SAU next year represent a random sample of all ACT takers, how many students would you predict will be placed into MATH 099?
12. Suppose SAU wants to place the top 7% of students into an honors program. What ACT score should be used so that no more than 7% of students are placed into the honors program?

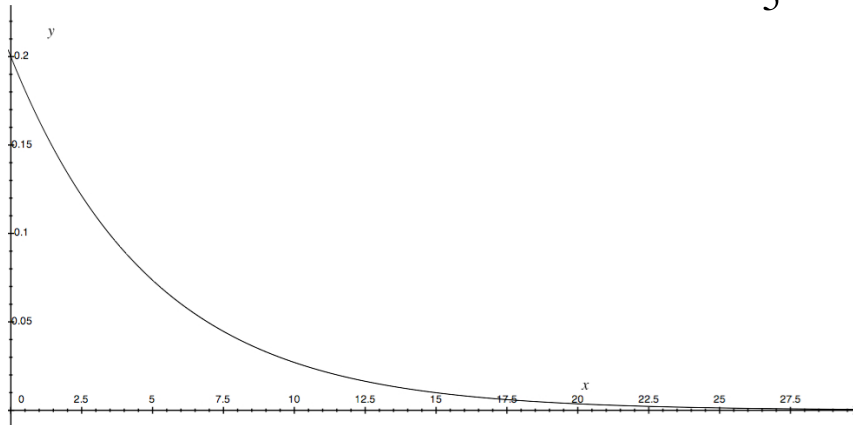
* Normal blood potassium levels range from 3.5-5.0 milliEquivalents per liter. Levels above 7.0 indicate severe hyperkalemia

Use the following information to answer questions 13-16:

Suppose the IT Help Desk at SAU waits, on average, 5 minutes between phone calls. With this information, we can model the time between calls with an exponential distribution with:

$$E[x] = \mu = \frac{1}{\lambda} = 5 \quad \lambda = \frac{1}{5} = 0.20 \quad \sigma^2 = \frac{1}{\lambda^2} = \frac{1}{\left(\frac{1}{5}\right)^2} = 25 \quad \sigma = 5$$

The probability function for this exponential distribution would be: $f(x) = \lambda e^{-\lambda x} = \frac{1}{5} e^{-\frac{x}{5}}$



13. What's the probability that the IT Help Desk waits more than 10 minutes for the next call?
14. If we repeatedly take independent, random samples of size 5 from this distribution and calculate the mean wait time for each sample, what would the sampling distribution (distribution of those means) look like?
15. Sketch the sampling distribution of means we would get if we repeatedly take independent, random samples of size 100 from this distribution?
16. Suppose you randomly sample 100 observations from the above distribution and calculate the average wait time. What's the probability that the average wait time for those 100 calls will be greater than 6 minutes? Find the 5th and 95th percentiles for the average wait time for 100 calls. What do those values represent?
17. Express the Central Limit Theorem in your own words. When does it apply? What is a sampling distribution? What is a standard error?