

## Activity #23: Dependent Samples Tests

You hypothesize that men tend to marry younger women. To test this hypothesis, you go to the county courthouse and record the ages appearing on marriage licenses for a sample of 15 couples. You collect the following data:

Couple	Husband	Wife
1	25	22
2	25	32
3	51	50
4	25	25
5	38	33
6	30	27
7	60	45
8	54	47
9	31	30
10	54	44
11	23	23
12	34	39
13	25	24
14	23	22
15	19	16

We could analyze this data in a number of ways:

1. Independent samples t-test
2. Dependent sample t-test
3. Nonparametric test (such as the sign test or signed ranks test)

In order to run an independent samples t-test, we must make the following assumptions:

1. Our data are independent (there are no links or pairings across groups)
2. Our populations have equal variances
3. The ages of husbands and wives in our population are each normally distributed

Obviously, it wouldn't be wise to blindly make these assumptions with this data set:

1. A wife's age is dependent upon the husband's age (if we're given some information about the age of the husband, our prediction for the age of the wife changes)
2. The variance assumption isn't necessarily satisfied, either. The husband ages have a variance of 185, while the wife ages have a variance of only 113. Using SPSS, I ran a hypothesis test to test the following hypotheses:

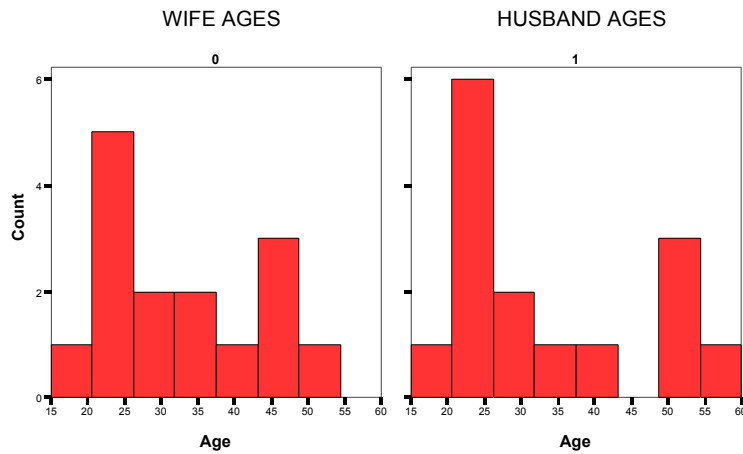
$$H_0 : \sigma_H^2 = \sigma_W^2$$

$$H_1 : \sigma_H^2 \neq \sigma_W^2$$

The p-value calculated by this test was  $p = 0.297$ . Therefore, I retain the null hypothesis and conclude the variances for the populations are not significantly different.

3. The normality assumption can be ignored if our sample size is large. Since our sample size is only  $n=15$  for each group, we'll have to make a judgment call about normality. Here are a couple tests SPSS can run to test for normality:

The "Sig." column reports p-values. According to both tests, it appears as though WIFE AGES are normally distributed whereas HUSBAND AGES are significantly different from normality. To see this, we could create histograms of the two samples...

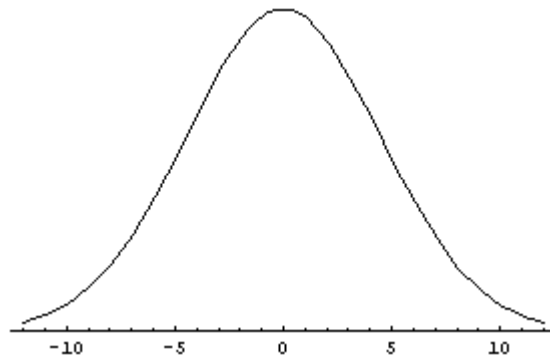


Even though we have dependent data, let's see what happens when we conduct an independent samples t-test.

**INDEPENDENT SAMPLES T-TEST** (using a 0.05 level of significance)

Hypotheses:  $H_0 : \mu_H - \mu_W = 0$   
 $H_1 : \mu_H - \mu_W > 0$

Sampling distribution will be centered at zero with a standard error of:  $\sqrt{\frac{1}{15} + \frac{1}{15}} \sqrt{\frac{14(13.6)^2 + 14(10.7)^2}{(14) + (14)}} = 4.47$



Our critical value, from the table in our textbook, is  $t_{28,0.05} = 1.701$

We can convert this to the  $\bar{X}$ -scale:  $(1.701)(4.47) + 0 = 7.603$

Our observed value is:  $\bar{X}_H - \bar{X}_W = 34.47 - 31.93 = 2.54$

We can convert this to the  $t$ -scale:  $t_{28} = \frac{2.54 - 0}{4.47} = 0.57$

Our decision is to retain the null hypothesis. The p-value is calculated to be approximately 0.30

Husband	Wife	Difference
25	22	2
25	32	-7
51	50	1
25	25	0
38	33	5
30	27	3
60	45	15
54	47	7
31	30	1
54	44	10
23	23	0
34	39	-5
25	24	1
23	22	1
19	16	3

Since we have dependent data, we obviously should not run an independent samples t-test (we would like the increase in power that comes with the appropriate application of a dependent samples t-test.) We still have to decide whether or not to run a dependent samples t-test.

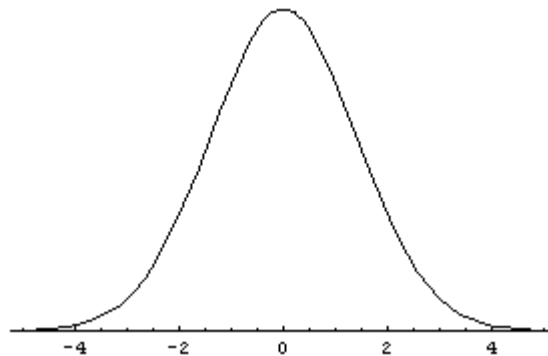
In order to run the dependent samples t-test, our data still must be normally distributed. We could argue that since our data is somewhat close to a normal distribution, the CLT will kick-in even though we have a relatively small sample size. Note: In real statistical analyses, we tend to have hundreds or thousands of observations, so this normality assumption can be made without thought.

### DEPENDENT SAMPLES T-TEST (using a 0.05 level of significance)

Hypotheses:  $H_0 : \mu_D = 0$   
 $H_1 : \mu_D > 0$

Statistics calculated from the difference scores:  $\bar{X}_d = \bar{d} = 2.53$  and  $s_d = 5.4$

Sampling distribution will be centered at zero with a standard error of:  $SE = \frac{SD}{\sqrt{n}} = \frac{5.4}{\sqrt{15}} = 1.4$



Our critical value, from the table in our textbook, is  $t_{14,.05} = 1.761$

We can convert this to the  $\bar{X}$ -scale:  $(1.761)(1.4) + 0 = 2.465$

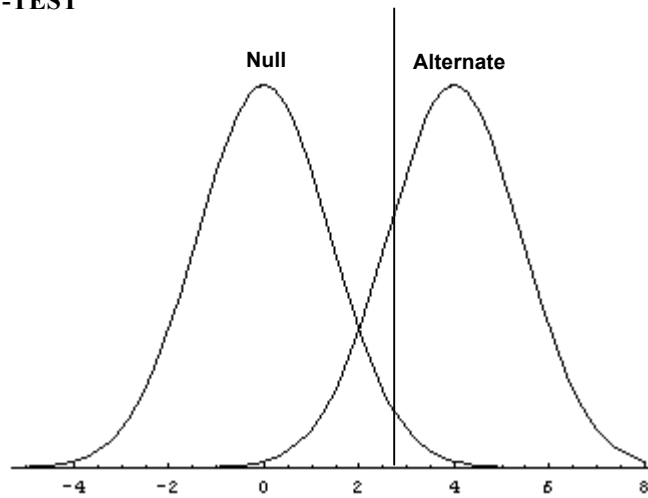
Our observed value is:  $\bar{X}_D = 2.53$

We can convert this to the  $t$ -scale:  $t_{14} = \frac{2.53 - 0}{1.4} = 1.807$

Our decision is to reject the null hypothesis. The p-value is calculated to be approximately 0.0455

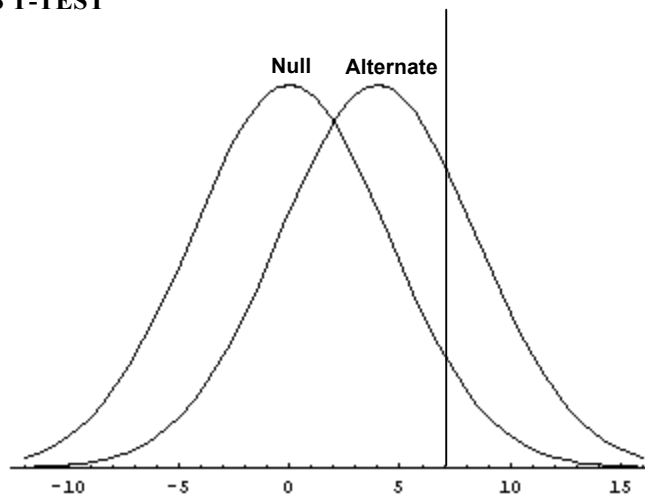
**Power Calculations (assuming the true difference between husband and wife ages is 4 years)**

**DEPENDENT SAMPLES T-TEST**



$$\text{Power: } P(\bar{X}_d > 2.465) = P(t_{14} > \frac{2.465 - 4}{1.4}) = P(t_{14} > -1.1) \approx 0.85$$

**INDEPENDENT SAMPLES T-TEST**



$$\text{Power: } P(\bar{X}_H - \bar{X}_W > 7.603) = P(t_{28} > \frac{7.603 - 4}{4.41}) = P(t_{28} > 0.817) \approx 0.21$$

Husband	Wife	Sign
25	22	+
25	32	-
51	50	+
25	25	<b>0</b>
38	33	+
30	27	+
60	45	+
54	47	+
31	30	+
54	44	+
23	23	<b>0</b>
34	39	-
25	24	+
23	22	+
19	16	+

If we're unwilling to make the normality assumption, we must run a nonparametric test. Nonparametric tests typically have very low power.

From our data, we observe 11 (+'s), 2 (-'s), and 2 zeros.

Under the null hypothesis, we have a 50% chance of observing a + for any pair of observations.

### SIGN TEST

What is the probability of observing 11 (+'s) out of 13 pairs of observations (we'll ignore the zeros, which lowers power even more)?

Each pair of observations constitutes a Bernoulli random variable. Each pair is also independent of all other pairs, so we have a binomial distribution here.

What is the probability of observing 2 or fewer (-'s) in 13 cases?

$$P(X \leq 2 | \text{bin}(13, 0.5)) = P(X = 0) + P(X = 1) + P(X = 2) = 0.011$$

This represents our p-value, so we reject the null hypothesis.