Unit 1: Practice Problems

Solve as many of these as you can. (*I've provided some hints in parentheses*)
You may work together, but show all your work.

1. A car repair can be performed either on time or late and either satisfactorily or unsatisfactorily.
   The probability of a repair being on time and satisfactory is 0.26.
   The probability of a repair being on time is 0.74.
   The probability of a repair being satisfactory is 0.41.

   What is the probability of a repair being late and unsatisfactory?

   (Although you can use probability formulas to solve this, I recommend drawing a Venn Diagram with a circle for "on time" and another circle for "satisfactorily." Then I recommend labeling as many probabilities as you can on the diagram)

2. The length, width, and height of a manufactured part are classified as being either within or outside specified tolerance limits. In a quality inspection:
   86% of the parts were found to be within the tolerance limits for width, but only
   80% of the parts are within the specified tolerance limits for all three dimensions.
   However, 2% of parts are within the tolerance limits for width and length but not for height,
   and 3% of parts are within the tolerance limits for width and height but not length.
   Moreover, 92% of parts are within the tolerance limits for either width or height or both width & height.

   If a part is within the tolerance limits for height, what is the probability that it will also be within the tolerance limits for width?

   If a part is within the limits for length and width, what is the probability that it will be within the tolerance limits for all three dimensions?

   (Here's another one that is easier if you draw a Venn Diagram)

3. Suppose you get a job inspecting toys imported from China. You decide to sample some toys from each shipment and test them for lead-based paint. If **at least two** of the toys you sample test positive for lead-based paint, you will reject the entire shipment.

   Suppose you receive one shipment of 100 toys. Unbeknownst to you, 20 of those toys contain lead-based paint. If you sample and test 10 toys from this shipment, what's the probability that you will incorrectly fail to reject shipment. (In other words, what's the probability that you will find 0 or 1 toys with lead-based paint in your sample?)

   (A hypergeometric approach will work here. You could also count the total number of ways in which you could sample 10 toys from 100 and the total number of ways you could sample 0 or 1 of the 20 dangerous toys)

4. Suppose I often come to work late and I need to sneak into my office without any administrators seeing me. I know that I can take two different routes to my office. If I take route A, I will walk by one administrator who has a 30% chance of seeing me. If I take route B, I will walk by two administrators who each have a 20% chance of seeing me. Assume the probabilities that these administrators see me are independent. I also know that since route A is shorter, I take it 75% of the time and I take route B 25% of the time.

   Given all of this information, what's the probability that an administrator will see me when I come into work late tomorrow?

   (Find the probability of getting through route A; then find the probability of getting through route B. Finally, use the law of total probability to find the probability of getting through on any randomly selected day)

5. A survey of customers showed that 10% were dissatisfied with plumbing jobs done in their homes. Half the complaints dealt with *Pete's Plumbing Company*, which does 40% of the plumbing jobs in the town.

   Find the probability that a customer will receive an unsatisfactory plumbing job, given *Pete's Plumbing Company* does the work.

   (This should be a relatively straightforward conditional probability problem. Assume that customers will always complain if they receive an unsatisfactory plumbing job.)

6. The probability of colorectal cancer in a certain population is 0.3%. If a person has colorectal cancer, the probability that the *haemoccult test* is positive is 50% (we call this the *sensitivity* of the test). If a person does not have colorectal cancer, the probability that he still tests positive is 3% (we call this the *false-positive* rate). What is the probability that a person from this population who tests positive actually has colorectal cancer?

   (This is a question that was posed to 24 medical doctors. Only 1 of the 24 doctors could give the correct answer)

   Source: Hoffrage, Ulrich & Gigerenzer G. (1998). Using natural frequencies to improve diagnostic inferences. Academic Medicine, 73, 538-540.