For each problem, identify the distribution of the random variable X, calculate its expected value, and answer the question.

Possible Distributions: Binomial, Hypergeometric, Poisson, Geometric, Negative Binomial, Uniform, Exponential, or Normal

1. Telephone receptionists answer hundreds of calls a day. In order to get their phone call answered, a caller has to be lucky enough to place a call at just the time when a receptionist has become free from a previous client. Suppose the chance of this happening is 0.1. Let X = the number of calls a person must make until a receptionist answers. What's the probability that a person will have to call exactly 3 times to get a receptionist to answer?

2. An Air Force intercept squadron consists of 16 planes that should always be ready for immediate launch. However, a plane's engines are troublesome and there is a probability of 0.25 that the engines of a particular plane will not start at a given attempt. Let X = the number of planes that will immediately become airborne if the squadron is scrambled. What is the probability that at least 15 planes will be ready for immediate launch?

3. Assume we have the same situation as the previous problem. This time, let X = the number of attempts needed to start a given plane. What is the probability that a plane will start in the first or second attempts?

4. A small lake contains 50 fish. One day, a fisherman catches ten and tags them so they can be recognized if they're caught again. He releases the tagged fish back into the lake. The next day, another fisherman catches four fish. Let X = the number of tagged fish he catches. Calculate the probabilities of this fisherman catching 0, 1, 2, 3, or 4 tagged fish. Sketch the pdf.

5. Suppose a company needs to hire three new workers and that each applicant interviewed has a probability of 0.6 of being found acceptable. Let X = the total number of applicants the company needs to interview. What is the probability that the company will have to conduct fewer than 5 interviews?

6. On average, there are about 25 imperfections in 100 meters of optical cable. Let X = the number of imperfections in 1 meter of cable. Calculate the probability that there are no imperfections in one meter of cable. What is the probability that there is no more than one imperfection in one meter of cable?

7. Milk is shipped to retail outlets in boxes that hold 16 containers. One particular box, which happens to contain 6 underweight containers, is opened for inspection and five containers are chosen at random. Let X = the number of underweight milk containers in the sample chosen by the inspector. What is the probability that the inspector will find no underweight containers?

8. A hospital emergency room accepts an average of about 47 bone fracture patients per week. Let X = the number of bone fracture patients arriving in a certain day. A hospital manager decides to allocate resources that are sufficient to cope with up to three bone fractures per day. What's the probability that on any given day, these resources will be adequate?

9. A computer generates random numbers between 0 and 1. Let X = 1 if the number is between (0.00, 0.25), X=2 if the number is between (0.25, 0.50), X=3 if the number is between (0.50, 0.75), and X=4 if the number is between (0.75, 1.00). Calculate the probability that a random variable will fall between (0.00, 0.50).

10. A team of underwater salvage experts sets sail to search the ocean floor for the wreckage of a ship that is thought to have sunk within a certain area. The captain's experience is that in similar situations, it has taken an average of 20 days to locate a wreck. Since the ocean is so large, not finding the wreckage in one area does not alter the chances of finding the wreckage in another area. Let X = the time, in days, taken to locate the wreckage. The captain will be paid a huge bonus if he can locate the wreckage within the first week. What is the probability that the captain will receive the bonus?

1. Geometric $E[x]=\frac{1}{p}=\frac{1}{.1}=10$ $P(X=3)=p(1-p)^{x-1}=.1(1-.1)^{3-1}=0.081$

2. Binomial $E[x]=np=16(.75)=12$ $P(X\geq 15)=1-P(X\leq 14)=1-binomcdf(16,.75,14)=.0635$

3. Geometric $E[x]=\frac{1}{.75}=1.33$ $P(X\leq 2)=P(X=0)+P(X=1)+P(X=2)=0.9375$

4. Hypergeometric $E[x]=\frac{nr}{N}=\frac{4(10)}{50}=.8$ $P(X=0)=\frac{\binom{10}{0}\binom{40}{4}}{\binom{50}{4}}=.397$

$$P(X=1)=\frac{\binom{10}{1}\binom{40}{3}}{\binom{50}{4}}=.429 \qquad P(X=2)=.152 \qquad P(X=3)=.021 \qquad P(X=4)=.001$$

5. Negative Binomial $E[x]=\frac{r}{p}=\frac{3}{.6}=5$ $P(X<5)=P(X\leq 4)=0.68256$

6. Poisson $E[x]=0.25=\lambda$ $P(X\leq 1)=\frac{e^{-.25}(.25)^0}{0!}+\frac{e^{-.25}(.25)^1}{1!}=.9735$

7. Hypergeometric $E[x]=\frac{5(6)}{16}=.1.875$ $P(X=0)=\frac{\binom{6}{0}\binom{10}{5}}{\binom{16}{5}}=..0577$

8. Poisson $E[x]=6.71=\lambda$

$$P(X\leq 3)=\frac{e^{-6.71}(6.71)^0}{0!}+\frac{e^{-6.71}(6.71)^1}{1!}+\frac{e^{-6.71}(6.71)^2}{2!}+\frac{e^{-6.71}(6.71)^3}{3!}=.098$$

9. Uniform $E[x]=0.50$ $P(X\leq 2)=0.83$

10. Exponential $E[x]=\frac{1}{.05}=20$ $P(X\leq 7)=1-e^{-.05(7)}=0.295$