Activity #9a (Continuous Distributions)

For over 10 years, you have been a loyal employee in the graphing calculator department of Texas Instruments. Your boss, Mr. Sliderule, visits your office one day:

Sliderule: Since you helped me out with that study earlier, I'm going to try to help you out. Rumor has it that management is looking to replace you with a robot

You: What? They can't do this to me!

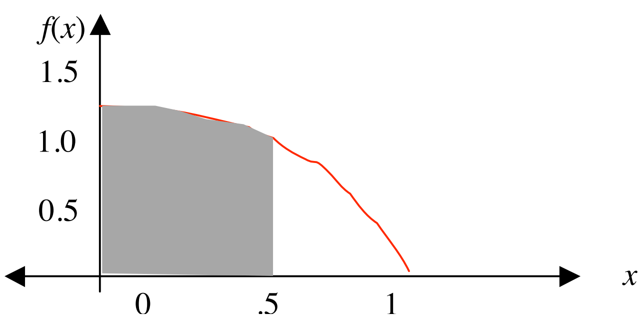
Sliderule: I know – I tried to change their minds, but they won't listen to reason.

You: Well, if they won't listen to reason, maybe they'll listen to statistics! Out of my way, Mr. Sliderule!

You push your boss out of the way and head straight down to the CEO's office. As you barge in, you see that the Industrial Robot salesman is already making his pitch. You hear him say that his robots are the most reliable in the business. In fact, the proportion of time X that one of his industrial robots is in operation during a 40-hour week is a random variable with pdf:

$$f(x) = -x^4 + \frac{6}{5} \quad \text{for } 0 \le X \le 1 \text{ (where X = percent of a 40-hour work week)}$$

1) Sketch this pdf. Use your graphing calculator (after all, you built it).



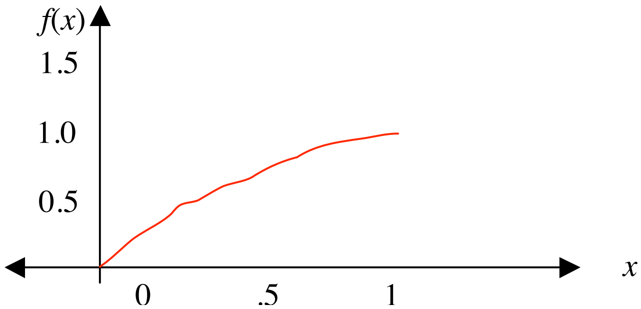
2) Let's make sure the Robot Salesman understands statistics. We know that a valid pdf satisfies two conditions:
(1) it yields positive probabilities for all possible values of X (no negative probabilities)
(2) the area under the curve sums to 1.0 (the probability of the entire sample space is 1.0)
Prove that this pdf satisfies both of these conditions.

Condition #1: Always positive. According to the graph, it passes this test

If we solve the inequality, we find that negative probabilities are yielded when X is greater than the square root of 6/5 (when X > 1.0467). Since this is outside the domain, we don't worry about it.

Condition #2: If we take the definite integral from 1 to 0, we do indeed find that it is equal to 1.0

3) Sketch the cdf for this random variable. Remember that a cdf is simply the indefinite integral of the pdf.



4) It looks as though this salesman does understand the distributions of continuous random variables. In order to save our job, we'll have to prove that we are more reliable than the robot. Let's say that the probability that you work 20 hours or less in a week is 0.15. Find the probability that the robot works 20 hours or less in a given week. Who is more likely to work less than half the week: the robot or you?

$$F(x) = \int_0^b (-x^4 + \frac{6}{5})dx = \left[\frac{-x^5}{5} + \frac{6x}{5}\right]_0^b$$

When b = 0.50, P(0≤X≤20) = 0.59375

5) Mr. Sliderule always says that to maintain maximum efficiency, a Texas Instruments employee must work at least 30 hours a week. Over the past 10 years, you have worked at least 30 hours 70% of the time. Calculate the probability that the robot works more than 30 hours in a given week. Who is more likely to work at least 30 hours a week: you or the robot?

$$F(x) = \int_{0.75}^{1.0} (-x^4 + \frac{6}{5})dx = \left[\frac{-x^5}{5} + \frac{6x}{5}\right]_{0.75}^{1.0} = 1.0 - 0.85 = 0.14746$$

6) Find the median number of hours that the robot functions. Interpret what the median represents in this situation.

$$0.5 = \int_0^k (-x^4 + \frac{6}{5})dx = \left[\frac{-x^5}{5} + \frac{6x}{5}\right]_0^k \qquad \left[\frac{-k^5}{5} + \frac{6k}{5}\right] = 0.5$$

$$\frac{6k - k^5}{5} = 0.5 \qquad 6k - k^5 = 2.5 \qquad 6k - k^5 - 2.5 = 0 \qquad k = 0.4188$$

I solved this using ROOT on the calculator after graphing it.
(0.4188)*(40) = under 17 hours. Half of the time, the robot only works 17 hours a week.

7) Find the mean (expected value) number of hours that the robot functions.

$$\int_{-\infty}^{\infty} x(-x^4 + \frac{6}{5})dx = \int (-x^5 + \frac{6x}{5})dx \qquad \left[\frac{-x^6}{6} + \frac{3x^2}{5}\right]_0^1 = (-\frac{1}{6} + \frac{3}{5}) = \frac{13}{31} = .4333$$

The mean number of hours worked per week = (.4333 * 40) = 17.333 (or 43% of the work week)

8) Using the results of your calculations (and anything else that might help), write a paragraph to the CEO of Texas Instruments. Explain to him why he shouldn't replace you with one of these industrial robots.

Half of the time, the robot works less than 17 hours per week.

It only operates 35+ hours a week 15% of the time.

59% of the time, it works less than 20 hours per week. I do that only 10% of the time.