Assignment #20: One-sample mean tests and confidence intervals

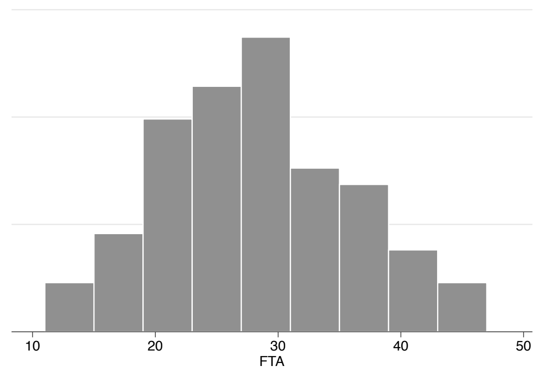
Scenario: A

During the 2010-11 NBA season, each team averaged just under 25 free throw attempts per game.

The Miami Heat averaged 27.902 free throw attempts per game that season:

Free Throw Attempts in each game

25	25	23	17	18	28	30	39	24
31	27	18	20	20	28	20	28	14
27	12	25	34	29	29	23	34	
34	38	21	37	26	23	19	22	
31	29	24	47	33	26	18	39	
24	29	38	19	22	37	30	30	
29	35	41	37	32	11	24	32	
47	42	18	32	28	27	28	21	
41	21	32	16	36	45	26	35	
36	21	25	30	19	22	29	26	



Summary: n = 82
average = 27.902
std. dev. = 7.881

We're going to conduct a test to determine if the Miami Heat attempted a significantly higher number of free throws than the rest of the league.

1) State the null and alternate hypotheses. Express the consequences of an alpha error in this study.

Null hypothesis: ______________________________

Alternate hypothesis: ______________________________

Alpha error consequence: ____

2) Why can't we simply look at the sample average, see it's higher than 25, and conclude the Miami Heat attempted more free throws per game than other teams?

3) What would be the more appropriate test in this situation: a z-test or a t-test? Explain. Conduct this test on your calculator and record the p-value.

4) Consider the assumptions necessary for us to conduct a t-test. Are these assumptions reasonably satisfied in this situation?

5) We're interested in μ (the population average number of free throws attempted by the Miami Heat). We'll never know the value of this parameter, but our best estimate is 27.902.

Suppose we could go back in time and play this 2010-11 season again. We'd expect the average free throws attempted by the Miami Heat would be different number (it wouldn't be exactly 27.902). Now suppose we go back in time again and again, each time recording the average number of free throws attempted by the Miami Heat. What would the sampling distribution of those averages look like? Sketch it below and label its center and spread.

6) Let's use $\alpha = 0.05$ in this study. Find the critical value of the t-statistic and shade-in the critical region of the distribution you sketched above. Convert this t-statistic to an average number of free-throws attempted.

Now, locate your observed average (27.902) on this distribution. From this, what's your decision regarding the null hypothesis? Do you reject or retain the null hypothesis? What's your conclusion regarding the number of free throws attempted by the Miami Heat? Estimate the p-value.

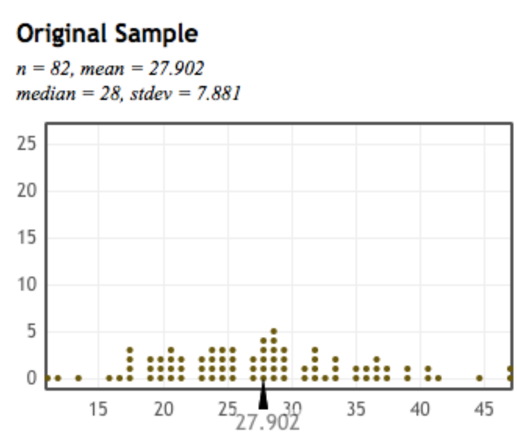
7) Suppose the true population average number of free throws attempted by the Miami Heat was 26. Estimate the power of this study.

To do this, it might be helpful to sketch (once again) the sampling distribution you sketched in question #5. Then, just to the right of that curve, sketch the alternate (true) distribution centered at 26. Thinking about what power represents, shade-in the power of this study and then calculate that power.

8) Let's construct a 90% confidence interval for the true population average number of free throws attempted by the Miami Heat. Before you construct this interval, can you predict whether it will contain 25?

9) If you completed the previous assignment (18b), you received solutions explaining a *bootstrap method* for constructing confidence intervals. The bootstrap method doesn't require a normality assumption like our standard (parametric) confidence interval method.

I'll demonstrate this bootstrap method using the following website:
http://lock5stat.com/statkey/bootstrap_1_quant/bootstrap_1_quant.html



First, I enter the data (our 82 free throw attempt values from page 1). You can see the data and summary statistics to the right.

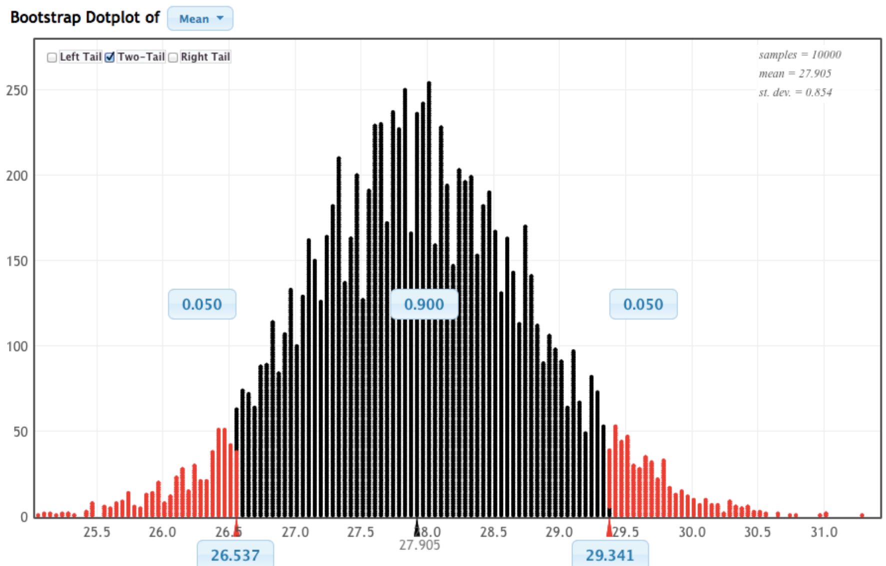
We'll now have the computer pretend these 82 values are the entire population. We'll instruct the computer to randomly select 82 values <u>with replacement</u> from this dataset and calculate and average.

We'll then have the computer repeat this process 10,000 times to give us an idea of the averages we could have obtained from our population.

On the next page, I've pasted the distribution of those possible averages.

10) Keep in mind what this graph represents. It shows possible averages we could have obtained from sample sizes of n = 82. It's centered at our observed sample average, since that's our best estimate of the population average.

To estimate our 90% confidence interval, I simply have the computer find the top and bottom 5% of our possible averages. As you can see below, the computer estimates this confidence interval to be (26.537, 29.341). How does this compare to the confidence interval you computed in question #8?



Remember our original question in this scenario: we're interested in determining if the Miami Heat attempted more free throws than the average NBA team (with 25 free throw attempts). We can use our bootstrap distribution to estimate just how unlikely it would be for the Miami Heat to average 25 (or fewer) free throw attempts.

Just by looking at the bootstrap distribution shown above, you can see it is extremely unlikely for the Miami Heat to average 25 or fewer free throw attempts per game.

11) Let's finish this example by running a sign test. Looking at sample of 82 free throw attempts (our original data on the first page), I see:

29 observations were less than 25 (so I will call these "–")
4 observations were exactly 25 (so I will ignore these)
49 observations were more than 25 (so I will call these "+")

So, in summary, out of 78 observations, we had 29 –'s and 49 +'s. Under a null hypothesis, we'd expect an equal number of +'s and –'s. Use the binomial distribution to estimate the likelihood of observing 29 or fewer –'s if, in fact, we'd expect an equal number.

Scenario: B	According to the International Diabetes Research Foundation, an individual has diabetes if their blood glucose concentration is at or above 200 mg/dl. Over the course of 3 months, you sample your blood 25 times and find the average glucose concentration is 201.38 with a standard deviation of 7.348 mg/dl. Test the hypothesis that you have diabetes based on these 25 measurements.

12) State the null and alternate hypotheses. Express the consequences of an alpha error in this study.

Null hypothesis: ______________________________________

Alternate hypothesis: ______________________________________

Alpha error consequence: ____

13) Why can't we simply look at the sample average, see it's higher than 200, and conclude we have diabetes?

14) What would be the more appropriate test in this situation: a z-test or a t-test? Explain. Conduct this test on your calculator and record the p-value.

15) Consider the assumptions necessary for us to conduct a t-test. Are these assumptions reasonably satisfied in this situation?

16) We're interested in μ (the population average blood glucose concentration). We'll never know the value of this parameter, but our best estimate is 201.38.

Suppose we could go back in time and get another sample of 25 measurements. We'd expect the average blood glucose level would be different number (it wouldn't be exactly 201.38). Now suppose we go back in time again and again, each time recording the average blood glucose level. What would the sampling distribution of those averages look like? Sketch it below and label its center and spread.

17) Let's use $\alpha = 0.05$ in this study. Find the critical value of the t-statistic and shade-in the critical region of the distribution you sketched above. Convert this t-statistic to an average blood glucose concentration.

Now, locate your observed average (201.38) on this distribution. From this, what's your decision regarding the null hypothesis? Do you reject or retain the null hypothesis? What's your conclusion regarding whether you have diabetes? Estimate the p-value.

18) Suppose your true population average blood glucose concentration was 202. Estimate the power of this study.

To do this, it might be helpful to sketch (once again) the sampling distribution you sketched in question #16. Then, just to the right of that curve, sketch the alternate (true) distribution centered at 202. Thinking about what power represents, shade-in the power of this study and then calculate that power.

19) Let's construct a 90% confidence interval for the true population blood glucose concentration. Before you construct this interval, can you predict whether it will contain 200?

Scenario C: In a discussion of SAT scores, someone comments:

Because only a minority of high school students take the SAT, the scores overestimate the ability of typical high school students. If all students took the SAT, the mean score would be no more than 450.

Suppose you give the SAT to 50 randomly sampled students in a high school. This sample may have included students who were or were not planning to take the SAT. From these 50 students, you find an average ACT score of 475 with a standard deviation of 97.

Conduct a t-test to determine if the average SAT score of all high school students is greater than 450.

a) Write your null and alternate hypotheses
b) Explain the consequences of an alpha error and choose the alpha level for your study
c) Briefly explain if you are concerned about any of the assumptions needed to conduct a t-test.
d) Sketch the sampling distribution, shade-in the critical region, and locate your observed data
e) Estimate the p-value and state your conclusion(s).
f) Assuming the true average SAT score for all high school students is 470, estimate power.

Scenario D: Do middle-aged male executives have high average blood pressure than the general population? The National Center for Health Statistics reports the average systolic blood pressure for males 35-44 years of age is 128. You get a random sample of medical records from 72 middle-aged male executives and calculate an average systolic blood pressure of 130 with a standard deviation of 15.

Conduct a t-test to determine if male executives have higher blood pressure than the general population.

a) Write your null and alternate hypotheses
b) Explain the consequences of an alpha error and choose the alpha level for your study
c) Briefly explain if you are concerned about any of the assumptions needed to conduct a t-test.
d) Sketch the sampling distribution, shade-in the critical region, and locate your observed data
e) Estimate the p-value and state your conclusion(s).
f) Assuming the true average SAT score for all high school students is 470, estimate power.

Scenario E: Your friend declares himself to be the world's greatest rock-paper-scissors player. You decide to challenge him for that title, so you play 510 times. Of those 510 trials, you lost 183 times (and either tied or won the other 327 times). Construct a 95% confidence interval to determine if your friend is better than average.

Scenario F: Evidence suggests that the drug Lipitor reduces total cholesterol. 1.9% of patients who used previously available cholesterol medications reported flu-like symptoms when taking the drug. In clinical trials, 11 out of 863 patients taking Lipitor complained of flu-like symptoms. Construct a 95% confidence interval to determine if Lipitor reduces the prevalence of flu-like symptoms.