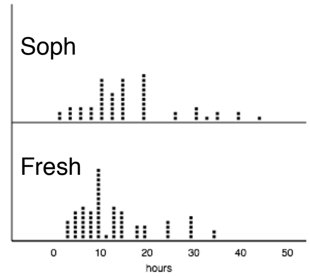
Example: Do sophomores spend more time studying than freshmen?

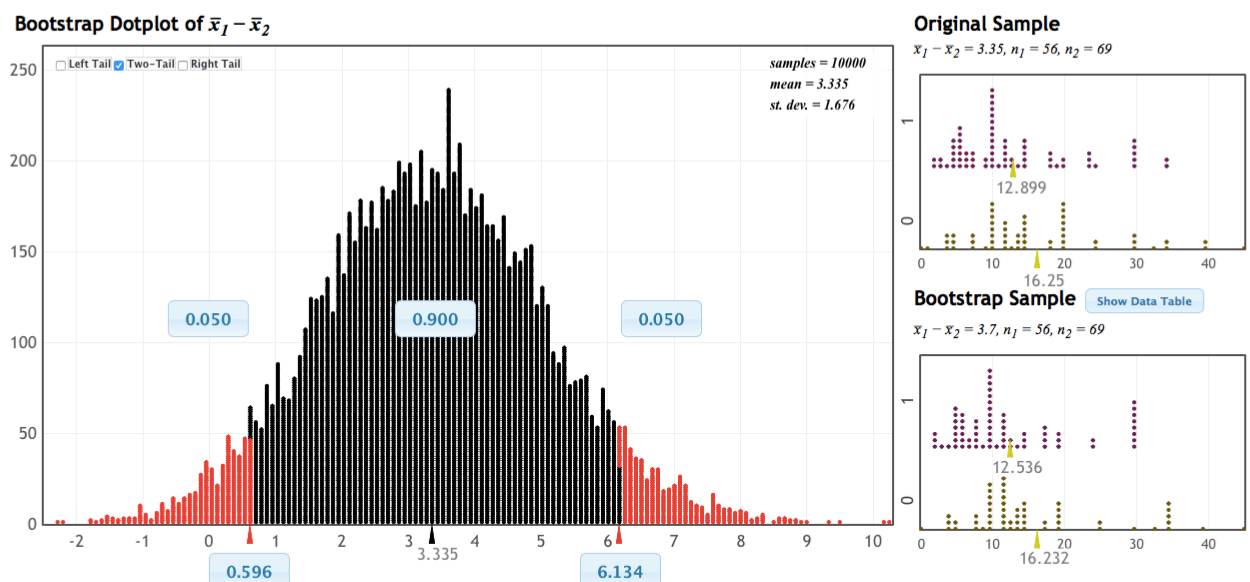
Data: Random samples of students who responded to a survey question.
Data can be downloaded at: http://www.bradthiessen.com/html5/stats/m300/hourstudy.txt

Summary:				
	Sophomores	**n = 56**	**mean = 16.25**	**std. dev = 10.268**
	Freshmen	**n = 69**	**mean = 12.90**	**std. dev = 8.293**
	Difference	**N = 125**	**mean diff = 3.35**	



Interval estimation: Goal = estimate the difference $\mu_{\text{soph}} - \mu_{\text{fresh}}$ with 90% confidence

Method 1: Bootstrap method
Applet: http://lock5stat.com/statkey/bootstrap_1_quant_1_cat/bootstrap_1_quant_1_cat.html

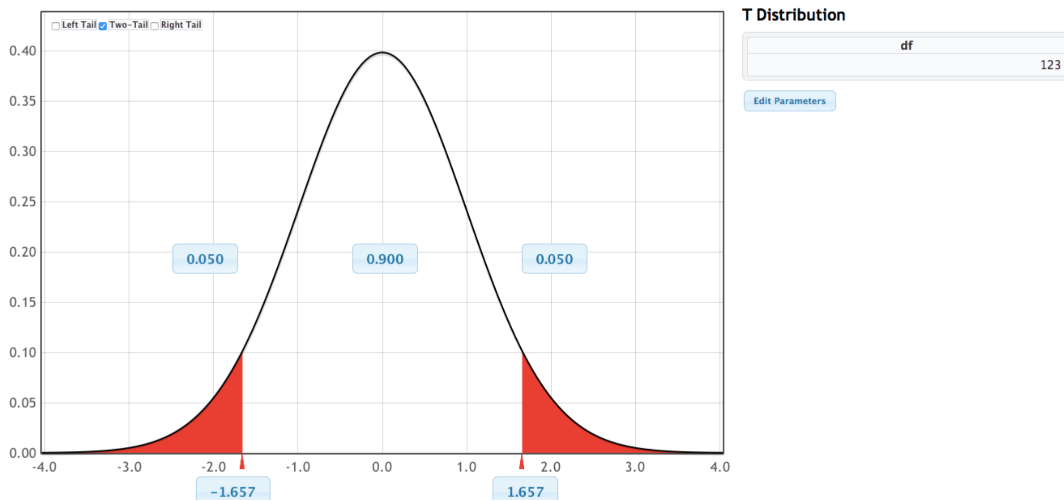


Result: 90% CI = (0.596, 6.134)

Method #2: Parametric (theory-based) method

Formula: $$\left(\bar{X}_s - \bar{X}_f\right) \pm \left(t_{n_s+n_f-2}\right)\sqrt{\frac{1}{n_s}+\frac{1}{n_f}}\sqrt{\frac{(n_s-1)s_s^2+(n_f-1)s_f^2}{n_s+n_f-2}}$$

Applet for t-distribution: http://lock5stat.com/statkey/theoretical_distribution/theoretical_distribution.html#t



Formula: $$(3.35) \pm (1.657)\sqrt{\frac{1}{56}+\frac{1}{69}}\sqrt{\frac{(56-1)(10.268)^2+(69-1)(8.293)^2}{56+69-2}}$$

$$= (3.35) \pm (1.657)(0.1799)(9.2285)$$

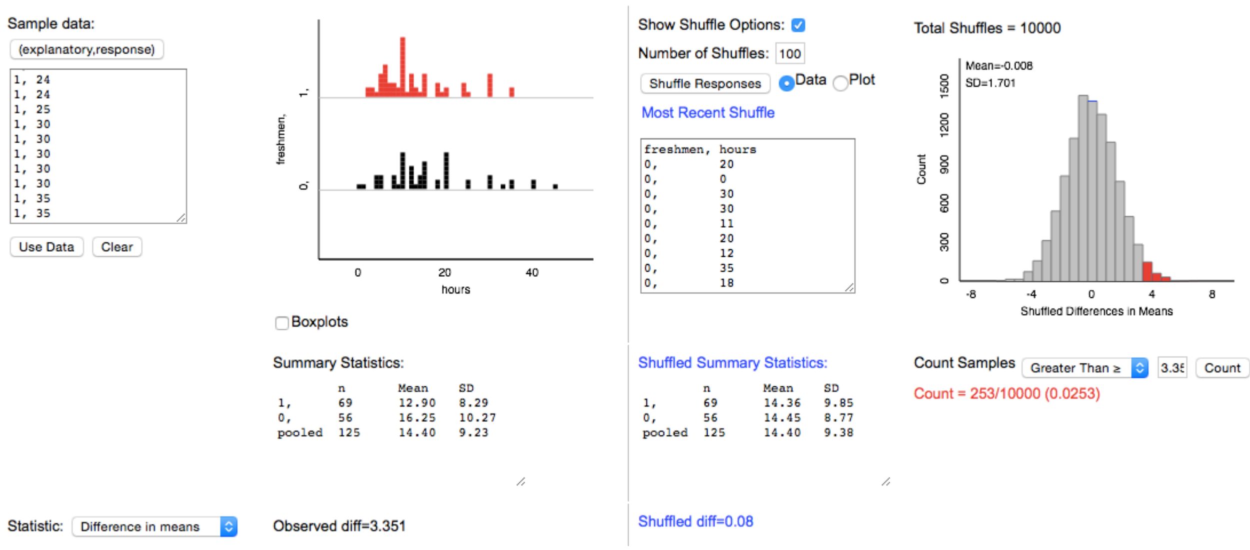
Result: 90% CI = (0.600, 6.102)   Note how similar this is to the CI from the bootstrap method

<u>Null hypothesis significance testing</u>: Goal = determine likelihood of observing data if the null hypothesis were true

Method 1: Randomization/Simulation method

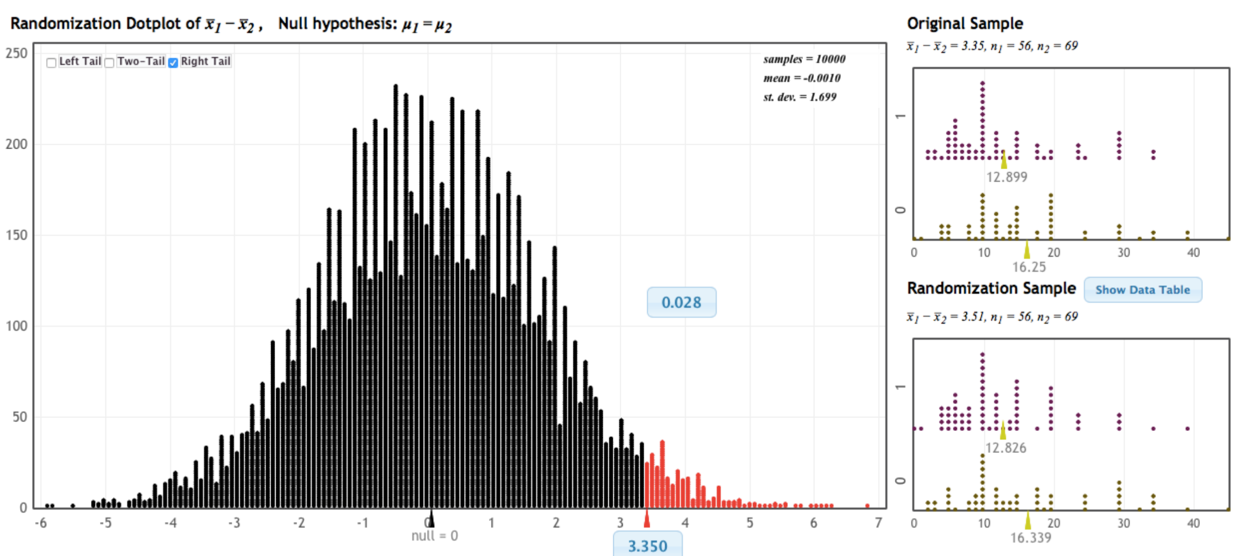
There are two applets that will do this. The first one animates the shuffling of groups.

Applet #1: http://www.rossmanchance.com/applets/AnovaShuffle.htm?hideExtras=2



Result: p-value = 0.0253

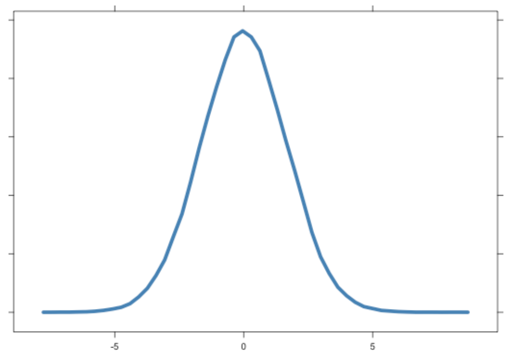
Applet #2: http://lock5stat.com/statkey/randomization_1_quant_1_cat/randomization_1_quant_1_cat.html



Result: p-value = 0.028

Method 2: Parametric (theory-based) method = Independent samples t-test

Sampling distribution:



Formula for standard error: $\sqrt{\frac{1}{56}+\frac{1}{69}}\sqrt{\frac{(56-1)(10.268)^2+(69-1)(8.293)^2}{56+69-2}}=1.66$

Critical value:

Applet: http://lock5stat.com/statkey/theoretical_distribution/theoretical_distribution.html#t

Result: t = 1.657 (same as what I did on page 2 of this example)

I can convert this critical t-value into a difference in averages using the formula:

$$(\mu_s-\mu_f)+\left(t_{n_s+n_f-2}\right)\left(SE_{pooled}\right)=(0)+(1.657)(1.66)=2.75$$

Observed value:

Result: The observed difference in averages is $\bar{X}_s-\bar{X}_f=3.35$

I can convert this observed difference in averages to a t-score using the formula:

$$t=\frac{(\bar{X}_s-\bar{X}_f)-(\mu_s-\mu_f)}{SE_{pooled}}=\frac{3.35-0}{1.66}=2.018$$

We can compare our observed and critical values to make a decision regarding the null hypothesis.

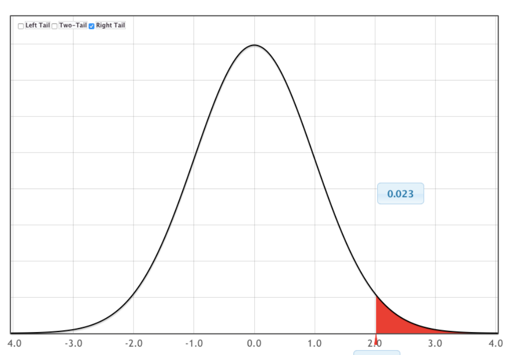
	Observed	Critical	
Difference in means	3.35	2.75	<– compare these
t-score	2.018	1.657	<- or compare these

Better yet, we can estimate a p-value:

p = P(observing our data or something more extreme | true null hypothesis)

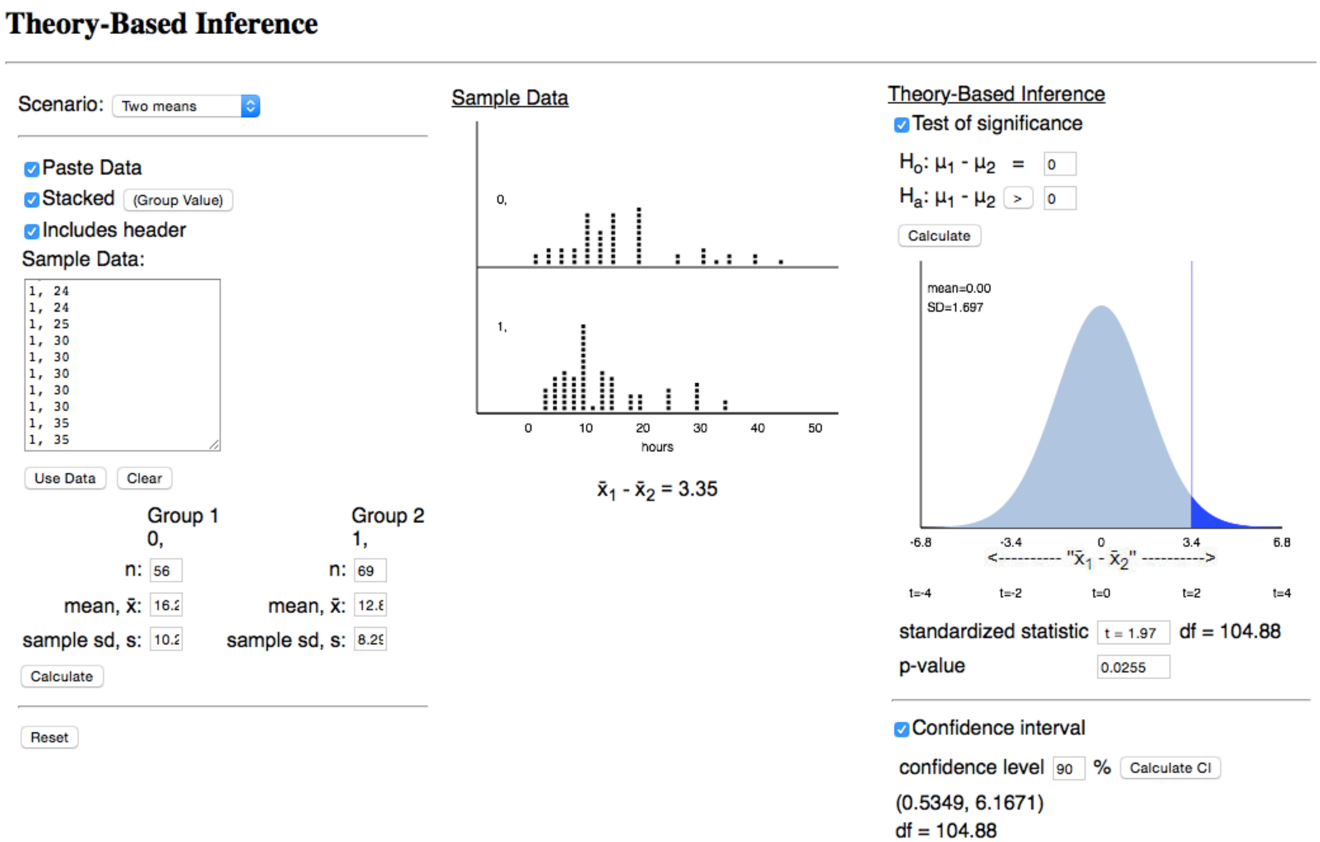
$p=P\left(\bar{X}_s-\bar{X}_f>3.35\right)=P\left(t_{123}>2.018\right)=0.023$ (see applet listed below to get this p-value)

Applet: http://lock5stat.com/statkey/theoretical_distribution/theoretical_distribution.html#t



Note that the following applet will run a complete independent samples t-test for you, but it does <u>not</u> make the equal variance assumption.

Applet: http://www.rossmanchance.com/applets/TBIA.html



The results (p = 0.0255 and a CI between 0.5349, 6.1671) are similar to what we obtained with our bootstrap, randomization, and parametric methods, but the degrees of freedom (df = 104.88) are weird.

This is because this applet is actually conducting a Welch–Satterthwaite test (which was mentioned at the end of activity #17).

I don't recommend you use this applet on the test, just because you won't be able to answer any follow-up questions (such as questions related to power). Also, if you're not willing to make the homogeneity of variances assumption, why not just use bootstrap/randomization methods?

Assuming the true (actual) difference in study time is $\mu_s - \mu_f = 3$, estimate the power of our t-test.

Power = P(rejecting the null hypothesis | the null hypothesis is false)
Looking at the sampling distribution and our critical value, this probability statement becomes:

$$P\left(\bar{X}_s - \bar{X}_f > 2.75 \;\middle|\; \mu_s - \mu_f = 3\right)$$

$$= P\left(t_{123} > \frac{2.75-3}{1.66}\right) = P\left(t_{123} > -0.15\right) = 0.559$$ (p-value was obtained with the t-distribution applet)