Unit 1 Assignment: Probability & Permutations

Source: Allan Rossman: http://statweb.calpoly.edu/arossman/stat325/notes.html

Suppose you only have 50 cents in your pocket and you want to buy an ice cream cone. The owner of the ice cream shop offers a random price determined as follows: You roll a pair of fair 6-sided dice and the price is the larger number followed by the smaller number (in cents).

What is the probability that you'll be able to afford the ice cream cone? **Answer: ______________________**

Method #1: Approximate through simulation

Let's see if we can simulate this situation. If you've downloaded and installed R on a computer (not a requirement), you can enter the following code. If you haven't installed R, just follow along.

Code:
```
d1 = sample(1:6,10000, replace=TRUE)
d2 = sample(1:6,10000, replace=TRUE)
price = 10*pmax(d1,d2) + pmin(d1,d2)
afford=(price<=50)
sum(afford)
```

Regardless of your background in programming, let's see if we can make sense of that code.

1. The first line records 10,000 random rolls of a 6-sided die. To do this, the code tells the computer to randomly sample 10,000 values (with replacement) from the integers 1-6. These values are recorded in a variable (think of it as a column in a spreadsheet) called d1.
2. The second line uses similar logic to simulate 10,000 rolls of our second 6-sided die. This variable is called d2.
3. The 3^{rd} line calculates the "price" we rolled in each of the 10,000 trials. To do this, it takes:
   10 x (the value of the die with the bigger number) + (the value of the die with the smaller number)
4. The 4^{th} line determines if we could have afforded the ice cream cone in each of our 10,000 trials. To do this, a variable named "afford" is created. We can afford the ice cream cone if, on any given trial, our "price" is less than 50.
5. The last line sums the number of times we could afford the ice cream (out of our 10,000 trials).

A) I ran this code in R and received the following output: sum(afford) = 4419. Based on this simulation, what's your best estimate of the probability that we could afford the ice cream cone?

Answer: ______________________

B) I ran this code 3 more times and received the following output: sum(afford) = 4361, sum(afford) = 4592, sum(afford) = 4461. Based on this additional information, what's your best estimate of the probability we could afford the ice cream?

Answer: ______________________

C) This time, I changed the first two lines of code so that only 100 dice rolls were simulated. I ran this new code and received the following output: sum(afford) = 40. What's your best estimate of the probability based on this output?

Answer: ______________________

D) Which of the above 3 answers gives you the "best" approximation? Why? ____

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Method #2: Exact numeration

Let's see if we can calculate the exact probability.

E) List all the possible outcomes from rolling a pair of fair, 6-sided dice. How many possible outcomes are there?

F) Looking at the sample space you just listed, circle the outcomes that represent the event that you can afford the ice cream. How many outcomes did you circle? From this, determine the (exact) probability that you can afford the ice cream cone.

Answer: ____________________

G) Is your answer to F) similar to your answers on the previous page?

Answer: ____________________

H) Suppose you are given a similar problem. Which method would you prefer to use to get probabilities: the simulation method or the exact calculation? Why? Identify a potential strength and weakness for each method.

The Mind Reader

I once served as a magician's assistant at *Playland Not-at-the-Beach* in El Cerrito, CA: http://www.playland-not-at-the-beach.org/
Based on this experience, I've come up with the following (lame) magic trick:

I ask an audience member to think of a 2-digit number, both digits must be unique odd numbers. I then guess his or her number and wait for applause.

Ok, so it's not the best trick in the world, but suppose I actually did this trick. In fact, suppose I had each of 1,000 audience members think of a 2-digit number (both digits must be unique odd numbers). Further suppose that when I stated my guess, 350 audience members admitted that I had guessed correctly.

I) If I had no magical abilities (and just guessed a number at random), what is the probability that I would have correctly guessed an individual's randomly-chosen number? How many audience members out of 1000 would have chosen that number at random?

J) I guessed 350 correctly out of 1000. Does this provide evidence that I have magical abilities? Identify some other plausible explanations for how I was able to guess the number for 350 audience members.

K) EXTRA CREDIT: Assuming every audience member chose a number at random and I guessed a number at random, what's the likelihood that I would have guessed 350 or more correctly out of 1000?