Activity #10: AxS ANOVA (Repeated subjects design)
Resources: optimism.sav

So far in MATH 300 and 301, we have studied the following hypothesis testing procedures:

1) Binomial test, sign-test, Fisher's method of randomization
2) One-sample t-test (comparing one mean to a hypothesized value)
3) Independent & Dependent samples t-tests (comparing two means)
4) z-test for proportions (comparing a proportions between groups)
5) One-way ANOVA (comparing two or more independent groups)
6) Two-Way ANOVA (comparing two or more independent groups across two factors)

We will now learn one more experimental design: the repeated measures (AxS ANOVA) design.

Using SAT analogies... <u>AxS ANOVA</u> : <u>ANOVA</u> :: <u>Dependent samples t-test</u> : <u>Independent samples t-test</u>

1) Fill-in the blanks to demonstrate your understanding of one-way ANOVA:

In a one-way ANOVA, the total sums of squares among observations is partitioned into two components:

____________________ (a measure of the variance due to the treatment effect), and

____________________ (a measure of the unexplained, random error, variance).

Sums of squares represent: ____

These sums of squares are then divided by their degrees of freedom to get ________________________

Mean squares represent: ____

We then compare the ratio of these mean squares to the _____ distribution.

Under the null-hypothesis, we assume this ratio to equal ______________________________________

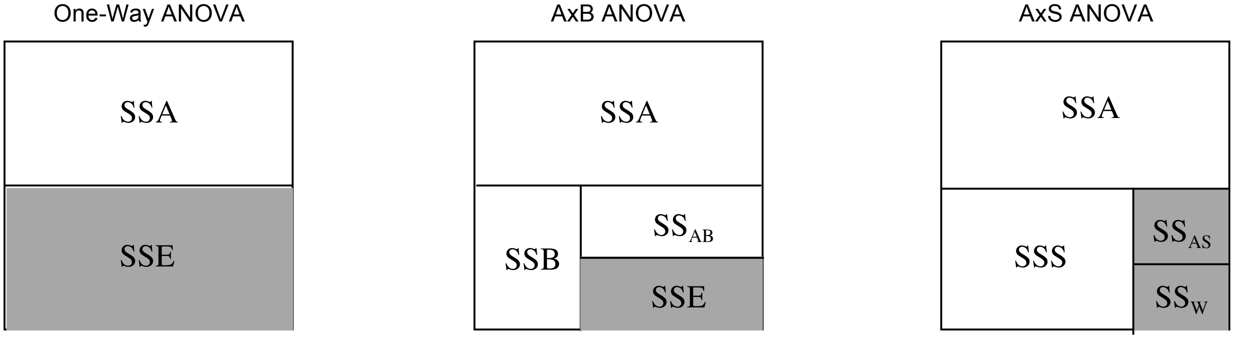
If the null hypothesis is false, we would expect the ratio of mean squares to be ______________________.

The power of a statistical hypothesis test refers to: ____

If we want to increase the power of our ANOVA, we could increase our sample size or increase α (the probability of a Type I error). Another way to increase the power of an ANOVA would be to decrease the size of MSE (the random variation within groups). If SSE is small, our F-ratio will be big and we will have a better chance of rejecting the null hypothesis.

The three diagrams below show how AxB and AxS ANOVA procedures attempt to reduce the size of MSE

One-Way ANOVA | AxB ANOVA | AxS ANOVA



Recall from MATH 300: *The dependent samples t-test is a more powerful test than the independent samples t-test, because it reduces the size of our standard error.*

2) Suppose we were interested in studying the effect of age on an individual's optimism. We could find three independent groups of subjects and set-up the completely randomized design as follows (higher numbers represent higher scores on an optimism test

	Age 16	Age 18	Age 20	Total
Subjects	18	18	18	54
Mean	25.9	26.9	23.4	25.4
Standard Deviation	6.58	7.49	5.92	

Source	SS	df	MS	Mean Square Ratio
SSA	120	2	60	1.34
SSE	2285	51	44.8	(Critical value = 3.18)
Total	2405	53	45.38	

From this one-way ANOVA, does it appear as though age has an impact on an individual's level of optimism?

3) Using the sums of squares values from the above summary table, calculate the percent of variance explained by the treatments (between-groups sums of squares).

4) Now suppose we conducted this study in a slightly different way. Instead of finding three independent groups of individuals, suppose we decide to track the same group of individuals for 4 years. This kind of design is called a *repeated measures* design (each subject is a member of each treatment). Do you see any reason why we would choose to conduct a *repeated measures* design instead of a simple one-way ANOVA? Are there any disadvantages to conducting this type of study.

5) The table on the top of the next page shows the data gathered from a repeated measures study. What does the last column represent?

Subject	Age 16	Age 18	Age 20	**Subject Means**
1	35	39	32	**35.3**
2	32	35	31	**32.7**
3	33	32	28	**31.0**
4	32	32	29	**31.0**
5	31	33	26	**30.0**
6	29	30	29	**29.3**
7	29	31	27	**29.0**
8	27	29	27	**27.7**
9	27	31	24	**27.3**
10	28	27	24	**26.3**
11	27	27	23	**25.7**
12	27	26	23	**25.3**
13	24	29	19	**24.0**
14	24	25	19	**22.7**
15	17	16	18	**17.0**
16	17	15	17	**16.3**
17	14	15	12	**13.7**
18	13	13	13	**13.0**
Means	**25.9**	**26.9**	**23.4**	**25.4**
Std. Dev.	**6.58**	**7.49**	**5.92**	

Notice that this data is identical to the data from the previous page:

	Age 16	Age 18	Age 20	
Subjects	18	18	18	54
Mean	25.9	26.9	23.4	25.4
Std. Dev	6.58	7.49	5.92	

5) Think about the sources of variation in this data. What are some reasons why two individuals in this study differ in optimism?

Some of the variance could be due to the effect of **age**. This is the treatment effect we're interested in estimating. If we were conducting a one-way ANOVA, all the other variation (within the columns) would be treated as random error (from unknown sources).

In this repeated measures design, we can partition this error variance further (explain some of it).

We know that some of the variation in optimism within each column is due to **pre-existing individual differences**. For example, subject #3 seems to have a tendency towards higher level of optimism and subject #18 has a tendency towards lower-levels of optimism. Since these "subject-effects" are now explained, we can subtract it from the unexplained error variance.

We may also expect that some variation in optimism is due to the interaction between a subject and his/her age (some people will become more optimistic; others will become less optimistic). We can subtract this "**subject interaction effect**" from the unexplained error variance.

Before we get to the calculations, take some time to think about what we're doing. We are saying that we can explain some of the variance within each treatment group through subject and subject-interaction effects. Because these sources of variation are explained, the remaining error variance shrinks. If MSE (the denominator of our ratio) is smaller, then our MSR will be larger, which will give us a greater probability of rejecting the null hypothesis (increase power).

6) Sketch how both ANOVA and AxS (repeated measures) ANOVA partition the total sums of squares.

ANOVA AxS ANOVA

7) The following table summarizes the calculations for an AxS ANOVA.

Source	Sums of Squares	Degrees of freedom	Mean Square	Mean Square Ratio
Between (Treatment)	$SS_A = \sum (p)(n)(\overline{X}_{A_j} - M)^2$	$a-1$	$\frac{SS_A}{df_A}$	
One-way ANOVA Error	$SS_E = \sum\sum (X - \overline{X}_{A_j})^2$	$an-a$	$\frac{SS_E}{df_E}$	
• **Subjects**	$SS_S = \sum (p)(a)(\overline{X}_{S_i} - M)^2$	$n-1$	$\frac{SS_S}{df_S}$	
• **AxS**	$SS_{AS} = \sum\sum (p)(\overline{X}_{S_i A_j} - \overline{X}_{S_i} - \overline{X}_{A_j} + M)^2$ or $SS_{AS} = SS_T - SS_A - SS_S - SS_W$	$(a-1)(n-1)$	$\frac{SS_{AS}}{df_{AS}}$	
• **Within Subject**	$\sum\sum\sum \left(X_{ijk} - \overline{X}_{s_i a}\right)^2$	$an(p-1)$	$\frac{SS_{WS}}{df_{WS}}$	
Total	$SS_T = \sum\sum (x_{ij} - M)^2$	$an-1$		

Notation: a = number of treatment groups (columns)
p = number of times a subject is measured within each group
n = the number of subjects within each treatment group (column)

Notes: The *One-way ANOVA Error* row is for reference only (conceptual reasons)
When p = 1, we will not have any within-subject variation

8) In this class, we will only be concerned with experiments in which subjects were measured only once in each treatment (p=1). In these cases, we can simplify the calculations:

Source	Sums of Squares	Degrees of freedom	Mean Square	Mean Square Ratio
Between (Treatment)	$SS_A = \sum (n)(\overline{X}_{A_j} - M)^2$	$a-1$	$\frac{SS_A}{df_A}$	
One-way ANOVA Error	$SS_E = \sum\sum (X - \overline{X}_{A_j})^2$	$an-a$	$\frac{SS_E}{df_E}$	
• **Subjects**	$SS_S = \sum (a)(\overline{X}_{S_i} - M)^2$	$n-1$	$\frac{SS_S}{df_S}$	
• **AxS**	$SS_{AS} = \sum\sum (\overline{X}_{S_i A_j} - \overline{X}_{S_i} - \overline{X}_{A_j} + M)^2$ or $SS_{AS} = SS_T - SS_A - SS_S - SS_W$	$(a-1)(n-1)$	$\frac{SS_{AS}}{df_{AS}}$	
Total	$SS_T = \sum\sum (x_{ij} - M)^2$	$an-1$		

Notation: a = number of treatment groups (columns)
n = the number of subjects within each treatment group (column)

Note: The *One-way ANOVA Error* row is for reference only (conceptual reasons)

9) Complete the summary table for this AxS ANOVA:

Source	Sums of Squares	Degrees of freedom	Mean Square	Mean Square Ratio
Between (Treatment)				
One-way ANOVA "Error"				
• **Subjects**				
• **AxS**				
Total				

Calculations:

$SS_A = 18[(25.9 - 25.4)^2 + (26.9 - 25.4)^2 + (23.4 - 25.4)^2] = 18(0.25 + 2.25 + 4) = 120$ *(same as ANOVA)*

$SS_E = \sum\sum(X - \overline{X}_{A_j})^2 = \sum(n-1)s_a^2 = (18-1)[(6.58^2 + 7.49^2 + 5.92^2)] = 2285$ *(same as one-way ANOVA)*

$SS_T = SS_A + SS_E = (35 - 25.4)^2 + (39 - 25.4)^2 + ... + (13 - 25.4)^2 = 2405$ *(same as one-way ANOVA)*

$SS_S = 3[(35.3 - 25.4)^2 + (32.7 - 25.4)^2 + ... + (13.7 - 25.4)^2 + (13.0 - 25.4)^2] = 2(726.43) = 2182$

$SS_{AS} = SS_E - SS_S = 2285 - 2182 = 103$

Notice $SS_E = SS_S + SS_{AS}$ (we partitioned the ANOVA error variance into two components)

10) Interpret the mean square values from the summary table. What are the expected values of those mean squares under a true null hypothesis? What are the expected values under a false null hypothesis?

MSA = average squared distance from the group means to the overall mean (treatment effect)
Expected value = $\sigma^2 + \alpha\eta + \alpha$

MSS = average squared distance from the subject means to the overall mean (subject effect)
Expected value = $\sigma^2 + \eta$

MSAS = variation due to the interaction between subjects and the treatments
Expected value = $\sigma^2 + \alpha\eta$

MSW = variation internal to each subject (since the subjects were measured only once per treatment, this is zero)
Expected value = σ^2

11) If we're interested in testing for a treatment effect (the effect of age on optimism), what mean square ratio should we use? Calculate this ratio and state your conclusion. Does this conclusion match our one-way ANOVA conclusion? Explain why the conclusions may differ.

12) Recall that in a one-way ANOVA, SSE (unexplained variance) was equal to 2285 (95% of the total variance). What is the value of SS_{AS} for our repeated-measures design? What percent of total variance is unexplained (due to error)? What percent of variance is due to pre-existing subject characteristics? What percent is due to the treatment?

13) What does the SS_{AS} represent in our repeated-measures ANOVA?

14) Suppose we calculated another MSR using MS_S and MS_{AS}. Do we expect this to be large or small? What would a high value of this MSR represent?

15) From our answers to the previous questions, we can see the advantage of the AxS ANOVA. As long as our subjects differ from one another (and stay somewhat consistent to themselves) across all treatments, we will observe a significant subject-effect. This means that our unexplained (error) variance will be smaller. This, in turn, means the denominator of our mean square ratio will be smaller. This causes our MSR to be larger, which makes it more likely that we'll reject the null hypothesis.

The value of $\frac{MS_S}{MS_{AS}}$ indicates how worthwhile it was for us to conduct an AxS ANOVA instead of a one-way ANOVA. If this mean square ratio is significant, then we eliminated a significant chunk of unexplained variance. If it is not significant, then we did not reduce the error variance (and we may have wasted our time).

We can measure the advantage in using AxS ANOVA by noting:

$$MSW_{ANOVA} = \frac{SS_S + SS_{AS}}{df_S + df_{AS}}$$

$MS_{AS} = MSW_{ANOVA}(1 - r_{pooled})$, where r is the correlation of subject scores across groups.

16) Let's write the formal model for the AxS ANOVA. Remember our model just identifies potential sources of variation in our data. Interpret each piece of the formal model.

One-Way ANOVA: $Y_{ij} = \mu + \alpha_j + \varepsilon_{ij}$

AxB ANOVA: $Y_{ijk} = \mu + \alpha_j + \beta_k + \alpha\beta_{jk} + \varepsilon_{ij}$

AxS ANOVA: $Y_{ijk} = \mu + \alpha_j + \eta_i + \alpha\eta_{ij} + \varepsilon_{ijk}$

17) What are the **disadvantages** of conducting an AxS ANOVA (rather than a one-way ANOVA)?

18) Optional discussion: Random effects vs. fixed effects models.
19) Optional discussion: Other experimental designs (GwT, AxR, AxBxC, Split-plot, Latin Square, etc.).

20) Complete the following AxS ANOVA example. Complete the summary table, state your conclusions, calculate an effect size, determine how worthwhile it was to run this as an AxS ANOVA, and describe any additional analyses you would run.

Situation: Cork deposits were collected on the north, east, south, and west sides of each of 5 trees. The researcher is interested in determining if the weight of the cork deposit is influenced by direction

Trees	North	South	East	West	**Tree Means**
1	30	28	16	34	**27**
2	14	18	10	22	**16**
3	24	20	18	30	**23**
4	38	34	20	44	**34**
5	26	28	14	30	**24.5**
Direction Mean	**26.4**	**25.6**	**15.6**	**32**	**24.9**
Std. Dev.	**8.76**	**6.54**	**3.85**	**8.00**	

Source	Sums of Squares	Degrees of freedom	Mean Square	Mean Square Ratio
Between (Treatment)				
One-way ANOVA "Error"				
• **Subjects**				
• **AxS**				
• *Within Subject*				
Total				