Activity #10b: Experimental Design Issues — Not in textbook

In the previous two activities, we learned how to conduct and interpret an AxB ANOVA. In this activity, we'll learn more about the assumptions necessary to run such an experimental design. We'll also learn another experimental design called the *Groups Within Treatments Design*.

Assumptions for AxB ANOVA:

1. Dependent variable must be a scale variable (continuous)
2. We have an independent random sample
3. The dependent variable is normally distributed within each treatment population
   a. P-P Plots; Histograms
   b. Normality Tests (Kolmogorov-Smirnoff, Chi-Square)
4. Homogeneity of variance
   a. Fmax using the largest and smallest cell variances
   b. Levene's Test
5. Proportionality
   a. Sample sizes must be equal or proportional in each cell
   b. If proportionality isn't met, $SS_{Total} \neq SS_A + SS_B + SS_{AB} + SS_E$
   c. Use the harmonic mean for each cell: $\tilde{n} = \frac{ab}{\sum 1/n_{jk}}$ and recalculate means

Proportional		
20	30	40
10	15	20

Proportional		
10	10	10
20	20	20

Proportional		
17	17	17
17	17	17

Not Proportional		
20	30	40
10	20	30

Not Proportional		
10	5	10
5	10	10

Groups Within Treatments Design

In one-way ANOVA, we assumed observations within each cell were independent (one didn't impact the other). Oftentimes, this independence assumption cannot be met. For example, assume we're interested in studying the effects of various textbooks on the achievement of statistics students. We could sample 3 classrooms and randomly assign them to use one of the textbooks. Our data would look like this:

Textbook A	Textbook B	Textbook C
Classroom #1 n = 16 students Mean, SD	Classroom #4 n = 10 students Mean, SD	Classroom #7 n = 5 students Mean, SD
Classroom #2 n = 20 students Mean, SD	Classroom #5 n = 15 students Mean, SD	Classroom #8 n = 10 students Mean, SD
Classroom #3 n = 30 students Mean, SD	Classroom #6 n = 14 students Mean, SD	Classroom #9 n = 30 students Mean, SD

On it's surface, this might appear to be a simple ANOVA (or possibly an AxB ANOVA). Notice that we only have one factor (textbook type) and that the observations within each treatment are grouped. That is why this is called a *Groups Within Treatments Design*.

1) Would you be willing to assume that all 66 observations within the Textbook A group are independent? Why or why not?

   No. Obviously, the teacher would create dependencies within each classroom.

2) How many observations do we have in this study: 150 students or 9 classrooms?

   It depends. If the groups are dependent, then we only have 9 observations. If the classrooms have no group dependency, then we have 150 observations.

Study: In Iowa, the local government may be organized in a variety of ways: mayor and council with all officers elected at large; mayor elected at large with council members elected by district; council members elected by district and mayor elected from within the council; variations of these organizational types with and without a hired city manager; etc. A political scientist wondered if one form of organization resulted in more "contented" citizens than another. He identified four types of government (A1, A2, A3, A4) to be studied. He then developed an interview-based scale by which a citizen could express his/her satisfaction with the local government's responsiveness to the needs and concerns of citizens. Five communities were identified that are organized under each type of government. Within each community, 50 heads of households were drawn at random by use of telephone directories, tax roles, water/sewer billings, and other sources. Each resident who could be located was interviewed to obtain a measure of overall satisfaction with the local government. The data are summarized below.

Govt. Type A1	**Govt. Type A2**	**Govt. Type A3**	**Govt. Type A4**
$n_{11} = 43$ $\bar{X}_{11} = 15.2$ $s^2_{11} = 50.3$	$n_{12} = 42$ $\bar{X}_{11} = 19.1$ $s^2_{11} = 42.5$	$n_{13} = 39$ $\bar{X}_{13} = 18.4$ $s^2_{13} = 52.6$	$n_{14} = 49$ $\bar{X}_{14} = 20.4$ $s^2_{14} = 51.0$
$n_{21} = 41$ $\bar{X}_{21} = 16.4$ $s^2_{21} = 42.8$	$n_{22} = 48$ $\bar{X}_{22} = 16.3$ $s^2_{22} = 48.7$	$n_{23} = 41$ $\bar{X}_{23} = 19.2$ $s^2_{23} = 48.2$	$n_{24} = 45$ $\bar{X}_{24} = 17.6$ $s^2_{24} = 43.4$
$n_{31} = 46$ $\bar{X}_{31} = 18.1$ $s^2_{31} = 49.6$	$n_{32} = 40$ $\bar{X}_{32} = 14.7$ $s^2_{32} = 50.1$	$n_{33} = 47$ $\bar{X}_{33} = 20.4$ $s^2_{33} = 40.4$	$n_{34} = 47$ $\bar{X}_{34} = 15.8$ $s^2_{34} = 40.5$
$n_{41} = 40$ $\bar{X}_{41} = 15.0$ $s^2_{41} = 38.3$	$n_{42} = 50$ $\bar{X}_{42} = 15.2$ $s^2_{42} = 35.3$	$n_{43} = 40$ $\bar{X}_{43} = 19.7$ $s^2_{43} = 38.2$	$n_{44} = 38$ $\bar{X}_{44} = 16.6$ $s^2_{44} = 44.2$
$n_{51} = 45$ $\bar{X}_{51} = 15.3$ $s^2_{51} = 45.9$	$n_{52} = 45$ $\bar{X}_{52} = 14.2$ $s^2_{52} = 48.4$	$n_{53} = 33$ $\bar{X}_{53} = 18.3$ $s^2_{53} = 42.7$	$n_{54} = 41$ $\bar{X}_{54} = 18.1$ $s^2_{54} = 56.9$

3) Notice how *stratified random sampling* was used. First, a sample of 5 communities within each government type was taken. Then, within each community, a sample of 50 individuals was taken. We have to ask ourselves the following question:

Do we have reason to believe the individuals (minor units) within each community (major unit) are similar to one another? In other words, do we believe there are dependencies within each community?

If the answer to this question is yes, we have to run the study with 20 observations (20 communities or major units)
If the answer to this question is no, we can run this study with 860 observations (860 individuals surveyed in all the communities)

Why would we want to run this study with 860 observations instead of 20 observations?

Higher sample size = more power

4) We will begin by examining the minor units (860 individuals). If we determine that significant dependencies exist within each major unit (community), we will have to run our analysis with only 20 observations. If we determine that there are no significant dependencies within each community, we'll run our analysis with 860 observations. The top of the next page shows the calculations used in our "minor unit analysis."

Note: A = Treatment Groups; K = Major Units (groups within each treatment)

Minor Unit Analysis				
Source	Sums of Squares	Degrees of freedom	Mean Square	Mean Square Ratio
Treatment (Government) (A)	$\sum n_{A_j}\left(\overline{X}_{A_j} - M\right)^2$	$a-1$	$\frac{SS_A}{df_A}$	
Between Groups Within Treatments (GwA)	$\sum\sum n_{K_j}\left(\overline{X}_{K_j} - \overline{X}_{A_j}\right)^2$	$K-a$	$\frac{SS_{GwA}}{df_{GwA}}$	$\frac{MS_{GwA}}{MS_W}$
Within Groups (W)	$\sum\sum\sum\left(X_{ikj} - \overline{X}_{K_j}\right)^2$ or $\sum(n_{K_j} - 1)s^2$	$N-K$	$\frac{SS_W}{df_W}$	

Here are the calculations based on our data. Note that weighted means are used in all calculations (weighted by the number of observations within each community).

Minor Unit Analysis				
Source	Sums of Squares	Degrees of freedom	Mean Square	Mean Square Ratio
Treatment (Government) (A)	1618.09	3	539.36	
Between Groups Within Treatments (GwA)	1655.79	16	103.49	2.28 (sig.)
Within Groups (W)	38187.9	840	45.64	

5) What do the various mean squares represent? What does our mean square ratio represent? Given that our mean square ratio is significant, what do we conclude?

A = distance between treatments

GwA = distance between the groups within each treatment (if there are no dependencies, we'd expect this to be small)

W = distance within each community (random error not due to community or government types)

MSR = how much of an impact community membership has on the dependent variable

We conclude that there are dependencies within each community, therefore we cannot treat all 860 observations as independent observations. We'll have to treat our communities as 20 random observations.

6) If we did not find significance, we could treat all 860 observations as random observations. To do this, we would just have to calculate Sserror = SSw + SSGwA and run a regular ANOVA.

7) Since we found a significant "groups within treatments" effect, we have to run a *major units analysis*. To do this, we need to recalculate the treatment means using unweighted means (ignore the sample size within each community). We then complete the following summary table:

Major Unit Analysis (following a significant GwA effect)				
Source	Sums of Squares	Degrees of freedom	Mean Square	Mean Square Ratio
Treatment (Government) (A)	$\sum k_j \left(\overline{X}^*_{A_j} - M^*\right)^2$	$a-1$	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{GwA}}$
Groups Within Treatments (GwA)	$\sum\sum \left(\overline{X}_{K_j} - \overline{X}^*_{A_j}\right)^2$	$K-a$	$\frac{SS_{GwA}}{df_{GwA}}$	

Here are the calculations based on our data. State the conclusion based on this analysis

Govt. Type A1	**Govt. Type A2**	**Govt. Type A3**	**Govt. Type A4**
$n_{11} = 43$ $\overline{X}_{11} = 15.2$ $s_{11}^2 = 50.3$	$n_{12} = 42$ $\overline{X}_{11} = 19.1$ $s_{11}^2 = 42.5$	$n_{13} = 39$ $\overline{X}_{13} = 18.4$ $s_{13}^2 = 52.6$	$n_{14} = 49$ $\overline{X}_{14} = 20.4$ $s_{14}^2 = 51.0$
$n_{21} = 41$ $\overline{X}_{21} = 16.4$ $s_{21}^2 = 42.8$	$n_{22} = 48$ $\overline{X}_{22} = 16.3$ $s_{22}^2 = 48.7$	$n_{23} = 41$ $\overline{X}_{23} = 19.2$ $s_{23}^2 = 48.2$	$n_{24} = 45$ $\overline{X}_{24} = 17.6$ $s_{24}^2 = 43.4$
$n_{31} = 46$ $\overline{X}_{31} = 18.1$ $s_{31}^2 = 49.6$	$n_{32} = 40$ $\overline{X}_{32} = 14.7$ $s_{32}^2 = 50.1$	$n_{33} = 47$ $\overline{X}_{33} = 20.4$ $s_{33}^2 = 40.4$	$n_{34} = 47$ $\overline{X}_{34} = 15.8$ $s_{34}^2 = 40.5$
$n_{41} = 40$ $\overline{X}_{41} = 15.0$ $s_{41}^2 = 38.3$	$n_{42} = 50$ $\overline{X}_{42} = 15.2$ $s_{42}^2 = 35.3$	$n_{43} = 40$ $\overline{X}_{43} = 19.7$ $s_{43}^2 = 38.2$	$n_{44} = 38$ $\overline{X}_{44} = 16.6$ $s_{44}^2 = 44.2$
$n_{51} = 45$ $\overline{X}_{51} = 15.3$ $s_{51}^2 = 45.9$	$n_{52} = 45$ $\overline{X}_{52} = 14.2$ $s_{52}^2 = 48.4$	$n_{53} = 33$ $\overline{X}_{53} = 18.3$ $s_{53}^2 = 42.7$	$n_{54} = 41$ $\overline{X}_{54} = 18.1$ $s_{54}^2 = 56.9$
$n_{A1} = 5$ $\overline{X}_{A1} = 16$	$n_{A2} = 5$ $\overline{X}_{A2} = 15.9$	$n_{A3} = 5$ $\overline{X}_{A3} = 19.2$	$n_{A4} = 5$ $\overline{X}_{A4} = 17.7$

Minor Unit Analysis				
Source	Sums of Squares	Degrees of freedom	Mean Square	Mean Square Ratio
Treatment (Government) (A)	36.9	3	12.3	5.28 (sig)
Between Groups Within Treatments (GwA)	37.34	16	2.33	