Assignment #8: Categorical Analysis

Source: Labby, Z. (2009). Weldon's dice, automated. *CHANCE*, 22(4), Fall 2009.
Another article discussing this data can be found at http://www.jstor.org/stable/2684294?seq=2

On February 2, 1894, Walter Frank Raphael Weldon wrote a letter to Francis Galton reporting the results of 26306 rolls of 12 dice. Weldon conducted his experiment to, "judge whether the differences between a series of group frequencies and a theoretical law, taken as a whole, were or were not more than might be attributed to the change fluctuations of random sampling."

In his experiment, Weldon rolled 12 dice and recorded the number of dice showing 5 or 6 dots. Thus, a die showing a 5 or 6 was defined to be a *success*. After recording the number of successes, Weldon repeated the process until he had rolled the 12 dice a total of 26306 times.

1. Before we look at the data, let's calculate some simple probabilities. If we roll one fair die, what's the probability we see 5 or 6 dots on that die?

2. Ok, that one was simple. These next probabilities, while still simple, require that you remember something basic from MATH 300. If we roll 12 fair dice, what's the probability we see no dice showing a 5 or 6? What's the probability we see exactly 1 die showing a 5 or 6? Calculate the probabilities and write them into the following table (under the P(# of successes) column).

Number of successes (# of dice showing 5 or 6 dots)	**P(# of successes)**	**Theoretical results**	**Observed results**		
0	__________	__________	185		
1	__________	__________	1149		
2	__________	__________	3265		
3	__________	__________	5475		
4	__________	__________	6114		
5	__________	__________	5194		
6	__________	__________	3067		
7	__________	__________	1331		
8	__________	__________	403		
9	__________	__________	105		
10	__________	__________	14		
11	__________	__________	4		
12	__________	__________	0		
SUM	1.00	26306	26306		

3. You should have calculated the probability of observing no dice showing 5 or 6 dots to be 0.007707. Now that you have those probabilities, calculate the numbers for the theoretical results column. This column represents the number of times (out of our 26306 trials) that we should have observed each result. Write these results into the table on the previous page. What assumption(s) are we making in calculating these theoretical results?

4. Weldon's actual results are displayed under the "observed results" column of the table. Obviously, the observed results do not match our theoretical results. I'd like to ask, "Why don't the observed results match the theoretical results?" but I know that question is too easy for you. I know you would say, "Either the dice are not fair or the results differ due to random sampling error."

   From our activities in class, we know how to conduct a Chi-Square goodness-of-fit test on this data to determine if the observed results differ significantly from the theoretical results. Conduct this test, report your calculated Chi-Square test statistic, and estimate the p-value. Briefly write any conclusions you can make from your results

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5. You should have found that the observed results differ significantly from the theoretical results. One proposed explanation is that the dice used in Weldon's experiment were cheap. Most inexpensive dice have hollowed-out pips representing the dots. Since opposite sides on a die have to add to 7, the side showing 5 or 6 would be lighter than the side showing 1 or 2 dots. Thus, we may expect 5s and 6s to appear slightly more frequently than expected. Is this explanation supported by the results you've calculated in the previous questions?

6. Let's suppose each die *is* slightly more likely to show 5 or 6 dots. If that's the case, how can we calculate the probability of one die showing a 5 or 6? When we could assume the dice were fair, it was easy to calculate: P(die shows 5 or 6) = 1/3. Now, with our extra knowledge about cheap dice, how can we estimate the probability?

   One way we could estimate this probability is by using Weldon's observed results.

   Since Weldon rolled 12 dice in each of his 26306 trials, we know he rolled a total of 12x26306 = 315,672 dice. We would like to know how many of these 315,672 dice showed 5 or 6 dots.

   The numbers in the "observed" results column represent the number of times he saw 0, 1, 2, …, or 12 dice (out of 12 possible) showing 5 or 6 dots. The numbers do **not** directly represent the number of dice showing 5 or 6 dots. Using the observed results in the table, find a way to calculate the total number of dice (out of 315,672) showing 5 or 6 dots.

7. Now use your result from the previous question to estimate the probability of one die showing a 5 or 6. Using this probability, we can now come up with new theoretical results for the experiment. Calculate these theoretical results and write them into the table below.

   Note: The rows for 10, 11, and 12 successes have been combined ("binned") into a single row ("bin") called, "10, 11, or 12." When conducting a chi-square goodness-of-fit test, you should combine groups like this so that every bin has a theoretical frequency of at least 4.

Number of successes (# of dice showing 5 or 6 dots)	**P(# of successes)**	**Theoretical results**	**Observed results**		
0	0.00712	187	185		
1	___________	___________	1149		
2	___________	___________	3265		
3	___________	___________	5475		
4	___________	___________	6114		
5	___________	___________	5194		
6	___________	___________	3067		
7	___________	___________	1331		
8	___________	___________	403		
9	___________	___________	105		
10, 11, or 12	.000612	16	18		
SUM	1.00	26306	26306		

8. From the results on the previous page, conduct a chi-square goodness-of-fit test and briefly write any conclusions you can make.

Unit #3 – Exercises

Reader's Digest conducted a study to find out how honest people are in different cities. Three cities of each type were selected: big cities, suburbs, medium cities, and small cities. In each selected city, 10 wallets were left in public places. Each wallet contained $50 cash, a telephone number, and an address where the owner could be reached. A record was kept of the number of wallets returned.

	Returned	**Kept Wallet**	**Total**
Big Cities	21	9	30
Suburbs	18	12	30
Medium Cities	17	13	30
Small Cities	24	6	30
Total	80	40	120

1. State your hypotheses and test for significant differences among the percentages of people who returned wallets in different types of cities.

2. Use the following data to test the hypothesis that a horse's chances of winning are unaffected by its position on the starting lineup. The data give the starting position of each of 144 winners, where position 1 is the closest to the inside rail of the racetrack.

Starting Position	1	2	3	4	5	6	7	8
# of wins	29	19	18	25	17	10	15	11

3. The drug Dramamine was tested for its effectiveness to prevent airsickness compared to a placebo. A total of 216 volunteers were randomly assigned to receive either the drug or the placebo. Of the 108 volunteers receiving treatment, 31 became airsick; of the 108 volunteers receiving the placebo, 60 became airsick. Create a contingency table to display these results and test whether Dramamine is effective in reducing the chances of airsickness.

4. Use the following data to test whether preferences for different formulations of a soft drink change with age.

	Formulation 1	**Formulation 2**	**Formulation 3**
Age 10-25	69	75	56
Age 26-50	82	64	54
Age 51 and over	74	84	42

5. Market researchers know that background music can influence the mood and purchasing behavior of customers. One study in a supermarket in Northern Ireland compared three treatments: no music, French accordian music, and Italian string music. Under each condition, the researchers recorded the numbers of bottles of French, Italian, and other wine purchased. Here is a summary of the data:

		Music			
		None	French	Italian	Total
Wine	French	30	39	30	99
	Italian	11	1	19	31
	Other	43	35	35	113
	Total	84	75	84	243