Section 2.1

Solutions to unit 1 online homework problems

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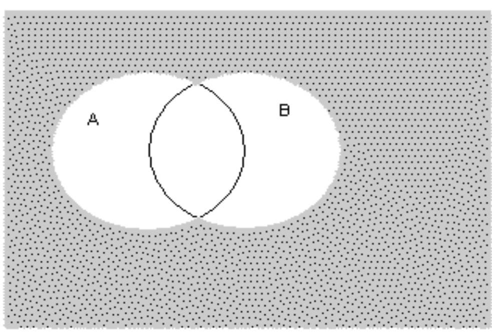
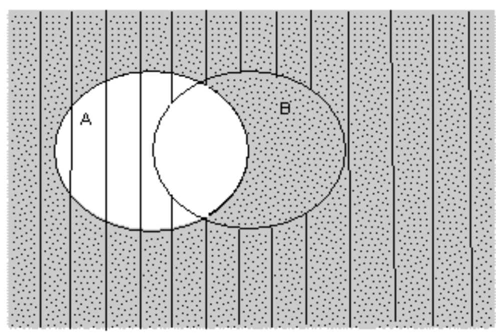
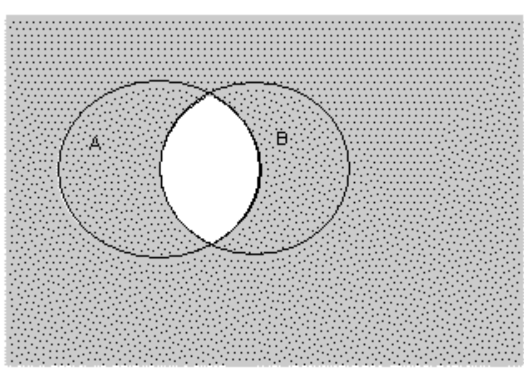
In the diagram on the left, the shaded area is (A∪B)′. On the right, the shaded area is A′, the striped area is B′, and the intersection A′ ∩ B′ occurs where there is BOTH shading and stripes. These two diagrams display the same area.





In the diagram below, the shaded area represents (A∩B)′. Using the diagram on the right above, the union of A′ and B′ is represented by the areas that have either shading or stripes or both. Both of the diagrams display the same area.



Section 2.2

11

f. .07

g. .15 + .10 + .05 = .30

h. Let event A = selected customer owns stocks. Then the probability that a selected customer does not own a stock can be represented by
$P(A') = 1 - P(A) = 1 - (.18 + .25) = 1 - .43 = .57$. This could also have been done easily by adding the probabilities of the funds that are not stocks.

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i. $P(A \cup B) = .50 + .40 - .25 = .65$

j. $P((A \cup B)') = 1 - P(A \cup B) = 1 - .65 = .35$

k. $A \cap B'$; $P(A \cap B') = P(A) - P(A \cap B) = .50 - .25 = .25$

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l. awarded either #1 or #2 (or both):
$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = .22 + .25 - .11 = .36$

m. awarded neither #1 or #2:
$P(A_1' \cap A_2') = P[(A_1 \cup A_2)'] = 1 - P(A_1 \cup A_2) = 1 - .36 = .64$

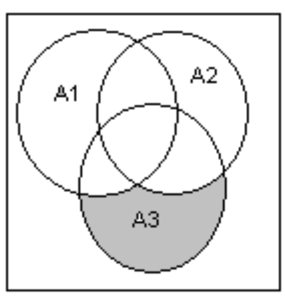
n. awarded at least one of #1, #2, #3:

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$
$$= .22 + .25 + .28 - .11 - .05 - .07 + .01 = .53$$

o. awarded none of the three projects:
$P(A_1' \cap A_2' \cap A_3') = 1 - P(\text{awarded at least one}) = 1 - .53 = .47$.

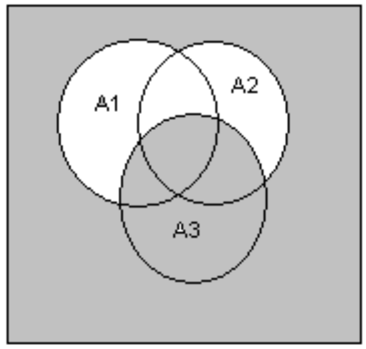
p. awarded #3 but neither #1 nor #2:

$$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$
$$= .28 - .05 - .07 + .01 = .17$$

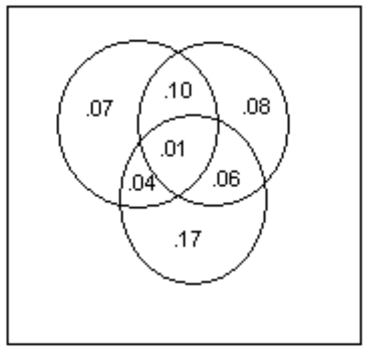


q. either (neither #1 nor #2) or #3:

$$P[(A_1' \cap A_2') \cup A_3] = P(\text{shaded region}) = P(\text{awarded none}) + P(A_3)$$
$$= .47 + .28 = .75$$



Alternatively, answers to **a – f** can be obtained from probabilities on the accompanying Venn diagram



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r. Let event E be the event that at most one purchases an electric dryer. Then E′ is the event that at least two purchase electric dryers.
$P(E') = 1 – P(E) = 1 - .428 = .572$

s. Let event A be the event that all five purchase gas. Let event B be the event that all five purchase electric. All other possible outcomes are those in which at least one of each type is purchased. Thus, the desired probability =
$1 – P(A) – P(B) = 1 - .116 - .005 = .879$

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t. The probabilities do not add to 1 because there are other software packages besides SPSS and SAS for which requests could be made.

u. $P(A') = 1 – P(A) = 1 - .30 = .70$

v. $P(A \cup B) = P(A) + P(B) = .30 + .50 = .80$
(since A and B are mutually exclusive events)

w. $P(A' \cap B') = P[(A \cup B)']$ (De Morgan's law)
$= 1 - P(A \cup B)$
$=1 - .80 = .20$

21

x. $P(\{M,H\}) = .10$

y. P(low auto) = P[{(L,N}, (L,L), (L,M), (L,H)}] = .04 + .06 + .05 + .03 = .18 Following a similar pattern, P(low homeowner's) = .06 + .10 + .03 = .19

z. P(same deductible for both) = P[{ LL, MM, HH }] = .06 + .20 + .15 = .41

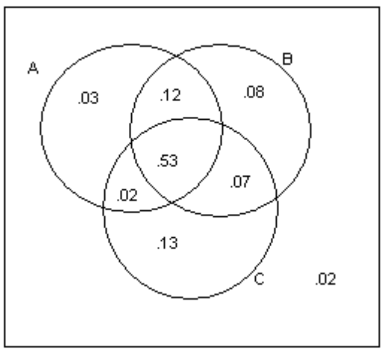
aa. P(deductibles are different) = 1 – P(same deductibles) = 1 - .41 = .59

bb. P(at least one low deductible) = P[{ LN, LL, LM, LH, ML, HL }]
= .04 + .06 + .05 + .03 + .10 + .03 = .31

cc. P(neither low) = 1 – P(at least one low) = 1 - .31 = .69

25 $(A \cap B) = P(A) + P(B) - P(A \cup B) = .65$
$P(A \cap C) = .55$, $P(B \cap C) = .60$

$$P(A \cap B \cap C) = P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C)$$
$$= .98 - .7 - .8 - .75 + .65 + .55 + .60$$
$$= .53$$



dd. $P(A \cup B \cup C) = .98$, as given.

ee. P(none selected) = 1 - $P(A \cup B \cup C)$ = 1 - .98 = .02

ff. P(only automatic transmission selected) = .03 from the Venn Diagram

gg. P(exactly one of the three) = .03 + .08 + .13 = .24

Section 2.3

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a. (26)(26) = 676; (36)(36) = 1296

b. $(26)^3 = 17{,}576$; $(36)^3 = 46{,}656$

c. $(26)^3 = 456{,}976$; $(36)^4 = 1{,}679{,}616$

d. $1 - 97{,}786/(36)^4 = .942$

30

e. Because order is important, we'll use $P_{8,3} = 8(7)(6) = 336$.

f. Order doesn't matter here, so we use $C_{30,6} = 593{,}775$.

g. From each group we choose 2: $\binom{8}{2} \bullet \binom{10}{2} \bullet \binom{12}{2} = 83{,}160$

h. The numerator comes from part c and the denominator from part b: $\dfrac{83{,}160}{593{,}775} = .14$

i. We use the same denominator as in part d. We can have all zinfandel, all merlot, or all cabernet, so P(all same) = P(all z) + P(all m) + P(all c) = $\dfrac{\binom{8}{6}+\binom{10}{6}+\binom{12}{6}}{\binom{30}{6}} = \dfrac{1162}{593{,}775} = .002$

31

j. $(n_1)(n_2) = (9)(27) = 243$

k. $(n_1)(n_2)(n_3) = (9)(27)(15) = 3645$, so such a policy could be carried out for 3645 successive nights, or approximately 10 years, without repeating exactly the same program.

33

l. $9! = 362{,}880$

m. 9! Starting line-ups and 9! batting orders → $(9!)(9!) = 131{,}681{,}894{,}400$

n. $\binom{5}{3} \times \binom{10}{6} = (10)(210) = 2{,}100$

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o. We want to choose all of the 5 cordless, and 5 of the 10 others, to be among the first 10 serviced, so the desired probability is $\dfrac{\binom{5}{5}\binom{10}{5}}{\binom{15}{10}} = \dfrac{252}{3003} = .0839$

p. Isolating one group, say the cordless phones, we want the other two groups represented in the last 5 serviced. So we choose 5 of the 10 others, except that we don't want to include the outcomes where the last five are all the same. So we have $\dfrac{\binom{10}{5}-2}{\binom{15}{5}}$. But we have three groups of phones, so the desired probability is $\dfrac{3\cdot\left[\binom{10}{5}-2\right]}{\binom{15}{5}} = \dfrac{3(250)}{3003} = .2498$.

q. We want to choose 2 of the 5 cordless, 2 of the 5 cellular, and 2 of the corded phones: $\dfrac{\binom{5}{2}\binom{5}{2}\binom{5}{2}}{\binom{15}{6}} = \dfrac{1000}{5005} = .1998$

41

r. P(at least one F among 1st 3) = 1 – P(no F's among 1st 3)

$$= 1 - \frac{4\times3\times2}{8\times7\times6} = 1 - \frac{24}{336} = 1 - .0714 = .9286$$

An alternative method to calculate P(no F's among 1st 3) would be to choose none of the females and 3 of the 4 males, as follows: $\dfrac{\binom{4}{0}\binom{4}{3}}{\binom{8}{3}} = \dfrac{4}{56} = .0714$, obviously producing the same result.

s. P(all F's among 1st 5) = $\dfrac{\binom{4}{4}\binom{4}{1}}{\binom{8}{5}} = \dfrac{4}{56} = .0714$

t. P(orderings are different) = 1 – P(orderings are the same for both semesters)
There are 8! possible orderings for the second semester, only one of which would match the first semester (whatever ordering that was). So, P(orderings are different) = 1 – 1/8! = .999975.

Section 2.4

45

u. P(A) = .106 + .141 + .200 = .447, P(C) =.215 + .200 + .065 + .020 = .500 P(A ∩ C) = .200

v. P(A|C) = $\dfrac{P(A \cap C)}{P(C)} = \dfrac{.200}{.500} = .400$. If we know that the individual came from ethnic group 3, the probability that he has type A blood is .40. P(C|A) = $\dfrac{P(A \cap C)}{P(A)} = \dfrac{.200}{.447} = .447$. If a person has type A blood, the probability that he is from ethnic group 3 is .447

w. Define event D = {ethnic group 1 selected}. We are asked for P(D|B′) = $\dfrac{P(D \cap B')}{P(B')} = \dfrac{.192}{.909} = .211$. P(D∩B′)=.082 + .106 + .004 = .192, P(B′) = 1 – P(B) = 1 – [.008 + .018 + .065] = .909

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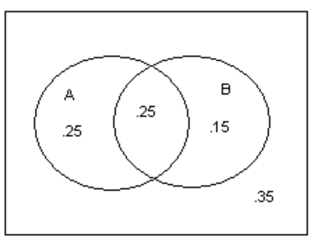


x. P(B | A) = $\dfrac{P(A \cap B)}{P(A)} = \dfrac{.25}{.50} = .50$

y. P(B′ | A) = $\dfrac{P(A \cap B')}{P(A)} = \dfrac{.25}{.50} = .50$

z. P(A | B) = $\dfrac{P(A \cap B)}{P(B)} = \dfrac{.25}{.40} = .6125$

aa. P(A′ | B) = $\dfrac{P(A' \cap B)}{P(B)} = \dfrac{.15}{.40} = .3875$

bb. P(A | A∪B) = $\dfrac{P[A \cap (A \cup B)]}{P(A \cup B)} = \dfrac{.50}{.65} = .7692$

49 The first desired probability is P(both bulbs are 75 watt | at least one is 75 watt).

P(at least one is 75 watt) = 1 – P(none are 75 watt) = 1 – $\dfrac{\binom{9}{2}}{\binom{15}{2}} = 1 - \dfrac{36}{105} = \dfrac{69}{105}$.

Notice that P[(both are 75 watt)∩(at least one is 75 watt)] = P(both are 75 watt) = $\dfrac{\binom{6}{2}}{\binom{15}{2}} = \dfrac{15}{105}$. So P(both bulbs are 75 watt | at least one is 75 watt) = $\dfrac{\frac{15}{105}}{\frac{69}{105}} = \dfrac{15}{69} = .2174$

Second, we want P(same rating | at least one NOT 75 watt).

P(at least one NOT 75 watt) = 1 – P(both are 75 watt)

$= 1 - \dfrac{15}{105} = \dfrac{90}{105}$.

Now, P[(same rating)∩(at least one not 75 watt)] = P(both 40 watt or both 60 watt).

P(both 40 watt or both 60 watt) = $\dfrac{\binom{4}{2}+\binom{5}{2}}{\binom{15}{2}} = \dfrac{16}{105}$

Now, the desired conditional probability is $\dfrac{\frac{16}{105}}{\frac{90}{105}} = \dfrac{16}{90} = .1778$

50

cc. P(M ∩ LS ∩ PR) = .05, directly from the table of probabilities

dd. P(M ∩ Pr) = P(M,Pr,LS) + P(M,Pr,SS) = .05+.07 = .12

ee. P(SS) = sum of 9 probabilities in SS table = .56, P(LS) = 1 - .56 = .44

ff. P(M) = .08+.07+.12+.10+.05+.07 = .49

P(Pr) = .02+.07+.07+.02+.05+.02 = .25

gg. P(M|SS ∩ Pl) = $\dfrac{P(M \cap SS \cap Pl)}{P(SS \cap Pl)} = \dfrac{.08}{.04+.08+.03} = .533$

hh. P(SS|M ∩ Pl) = $\dfrac{P(SS \cap M \cap Pl)}{P(M \cap Pl)} = \dfrac{.08}{.08+.10} = .444$

P(LS|M Pl) = 1 - P(SS|M Pl) = 1 - .444 = .556

53

P(B | A) = $\dfrac{P(A \cap B)}{P(A)} = \dfrac{P(B)}{P(A)}$ (since B is contained in A, A ∩ B = B)

$$= \frac{.05}{.60} = .0833$$

55

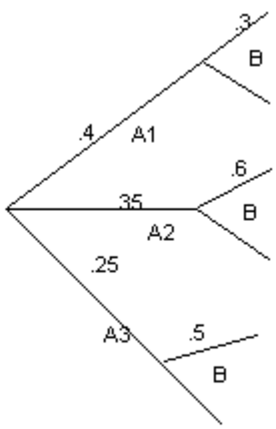
Let A = {carries Lyme disease} and B = {carries HGE}. We are told P(A) = .16, P(B) = .10, and $P(A \cap B \mid A \cup B) = .10$. From this last statement and the fact that $A \cap B$ is contained in $A \cup B$,

$.10 = \frac{P(A \cap B)}{P(A \cup B)} \rightarrow P(A \cap B) = .10P(A \cup B) = .10[P(A) + P(B) - P(A \cap B)] = .10[.10 + .16 - P(A \cap B)] \rightarrow 1.1P(A \cap B) =$ $.026 \rightarrow P(A \cap B) = .02364$.

Finally, the desired probability is $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.02364}{.10} = .2364$.

$.4 \times .3 = .12 = P(A_1 \cap B) = P(A_1) \bullet P(B \mid A)$

59



$.35 \times .6 = .21 = P(A_2 \cap B)$

$.25 \times .5 = .125 = P(A_3 \cap B)$

ii. $P(A_2 \cap B) = .21$

jj. $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .455$

kk. $P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{.12}{.455} = .264$, $P(A_2|B) = \frac{.21}{.455} = .462$, $P(A_3|B) = 1 - .264 - .462 = .274$

Section 2.5

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ll. Since the events are independent, then A′ and B′ are independent, too. (See paragraph below equation 2.7.) $P(B'|A') = .$ $P(B') = 1 - .7 = .3$

mm. $P(A \cup B)=P(A) + P(B) - P(A) \cdot P(B) = .4 + .7 - (.4)(.7) = .82$

nn. $P(AB'| A \cup B) = \frac{P(AB' \cap (A \cup B))}{P(A \cup B)} = \frac{P(AB')}{P(A \cup B)} = \frac{.12}{.82} = .146$

80

P(system works) = P(1 – 2 works ∪ 3 – 4 works)
= P(1 – 2 works) + P(3 – 4 works) - P(1 – 2 works ∩ 3 – 4 works)
= P(1 works ∪ 2 works) + P(3 works ∩ 4 works) – P(1 – 2) • P(3 – 4)
= (.9+.9-.81) + (.9)(.9) – (.9+.9-.81)(.9)(.9)
= .99 + .81 - .8019 = .9981

83

P(both detect the defect) = 1 – P(at least one doesn't) = 1 - .2 = .8

oo. P(1^{st} detects ∩ 2^{nd} doesn't) = P(1^{st} detects) – P(1^{st} does ∩ 2^{nd} does)
= .9 - .8 = .1
Similarly, P(1^{st} doesn't ∩ 2^{nd} does) = .1, so P(exactly one does)= .1+.1= .2

pp. P(neither detects a defect) = 1 – [P(both do) + P(exactly 1 does)]
= 1 – [.8+.2] = 0
so P(all 3 escape) = (0)(0)(0) = 0.

108

qq. P(walks on 4^{th} pitch) = P(1^{st} 4 pitches are balls) = $(.5)^4$ = .0625

rr. P(walks on 6^{th}) = P(2 of the 1^{st} 5 are strikes, #6 is a ball)
= P(2 of the 1^{st} 5 are strikes)P(#6 is a ball)
= $[10(.5)^5](.5)$ = .15625

ss. P(Batter walks) = P(walks on 4^{th}) + P(walks on 5^{th}) + P(walks on 6^{th})
= .0625 + $4(.5)^5$ + .15625 = .34375

tt. P(first batter scores while no one is out) = P(first 4 batters walk)
= $(.34375)^4$ = .014

Section 3.1

1.

S:	FFF	SFF	FSF	FFS	FSS	SFS	SSF	SSS
X:	0	1	1	1	2	2	2	3

7

a. Possible values are 0, 1, 2, …, 12; discrete

b. With N = # on the list, values are 0, 1, 2, … , N; discrete

c. Possible values are 1, 2, 3, 4, … ; discrete

d. $\{ x: 0 < x < \infty \}$ if we assume that a rattlesnake can be arbitrarily short or long; not discrete

e. With c = amount earned per book sold, possible values are 0, c, 2c, 3c, … , 10,000c; discrete

f. $\{ y: 0 < y < 14\}$ since 0 is the smallest possible pH and 14 is the largest possible pH; not discrete

g. With m and M denoting the minimum and maximum possible tension, respectively, possible values are $\{ x: m < x < M \}$; not discrete

h. Possible values are 3, 6, 9, 12, 15, … -- i.e. 3(1), 3(2), 3(3), 3(4), …giving a first element, etc,; discrete

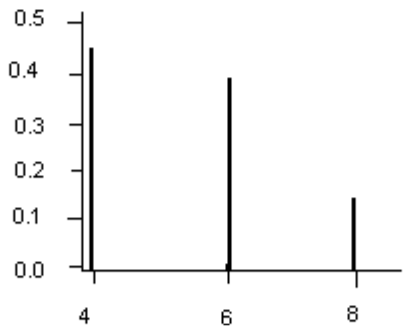
Section 3.2

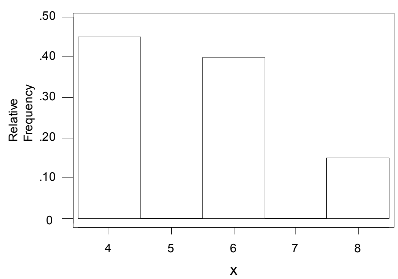
11

i.

x	4	6	8
P(x)	.45	.40	.15

j.





k. $P(x \geq 6) = .40 + .15 = .55$ $\quad$ $P(x > 6) = .15$

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l. $P(X \leq 3) = p(0) + p(1) + p(2) + p(3) = .10+.15+.20+.25 = .70$

m. $P(X < 3) = P(X \leq 2) = p(0) + p(1) + p(2) = .45$

n. $P(3 \leq X) = p(3) + p(4) + p(5) + p(6) = .55$

o. $P(2 \leq X \leq 5) = p(2) + p(3) + p(4) + p(5) = .71$

p. The number of lines not in use is $6 - X$, so $6 - X = 2$ is equivalent to $X = 4$, $6 - X = 3$ to $X = 3$, and $6 - X = 4$ to $X = 2$. Thus we desire $P(2 \leq X \leq 4) = p(2) + p(3) + p(4) = .65$

q. $6 - X \geq 4$ if $6 - 4 \geq X$, i.e. $2 \geq X$, or $X \leq 2$, and $P(X \leq 2) = .10+.15+.20 = .45$

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r. P(2) = P(Y = 2) = P(1st 2 batteries are acceptable)
= P(AA) = (.9)(.9) = .81

s. p(3) = P(Y = 3) = P(UAA or AUA) = $(.1)(.9)^2 + (.1)(.9)^2 = 2[(.1)(.9)^2] = .162$

t. The fifth battery must be an A, and one of the first four must also be an A. Thus, p(5) = P(AUUUA or UAUUA or UUAUA or UUUAA) = $4[(.1)^3(.9)^2] = .00324$

u. P(Y = y) = p(y) = P(the y^{th} is an A and so is exactly one of the first y – 1)
$=(y-1)(.1)^{y-2}(.9)^2$, y = 2,3,4,5,…

Section 3.3

29

v. $E(X) = \sum_{x=0}^{4} x \cdot p(x)$
= (0)(.08) + (1)(.15) + (2)(.45) + (3)(.27) + (4)(.05) = 2.06

w. $V(X) = \sum_{x=0}^{4} (x-2.06)^2 \cdot p(x) = (0-2.06)^2(.08) + \ldots + (4-2.06)^2(.05)$
= .339488+.168540+.001620+.238572+.188180 = .9364

x. $\sigma_x = \sqrt{.9364} = .9677$

y. $V(X) = \left[\sum_{x=0}^{4} x^2 \cdot p(x)\right] - (2.06)^2 = 5.1800 - 4.2436 = .9364$

31 E (Y) = .60;
$E(Y^2) = 1.1$
$V(Y) = E(Y^2) - [E(Y)]^2 = 1.1 - (.60)^2 = .74$
$\sigma_y = \sqrt{.74} = .8602$
E (Y) ± σ_y= .60 ± .8602 = (-.2602, 1.4602) or (0, 1).
P(Y = 0) + P(Y =1) = .85

44

z.

k	2	3	4	5	10
$\frac{1}{k^2}$	.25	.11	.06	.04	.01

aa. $\mu = \sum_{x=0}^{6} x \cdot p(x) = 2.64$, $\quad \sigma^2 = \left[\sum_{x=0}^{6} x^2 \cdot p(x)\right] - \mu^2 = 2.37, \sigma = 1.54$

Thus $\mu - 2\sigma = -.44$, and $\mu + 2\sigma = 5.72$,
so $P(|x-\mu| \ge 2\sigma) = P(X \text{ is at least 2 s.d.'s from } \mu)$
$= P(X \text{ is either} \le -.44 \text{ or} \ge 5.72) = P(X = 6) = .04$.
Chebyshev's bound of .25 is much too conservative. For k = 3,4,5, and 10, $P(|x-\mu| \ge k\sigma) = 0$, here again pointing to the very conservative nature of the bound $\frac{1}{k^2}$.

bb. $\mu = 0$ and $\sigma = \frac{1}{3}$, so $P(|x-\mu| \ge 3\sigma) = P(|X| \ge 1)$
$= P(X = -1 \text{ or } +1) = \frac{1}{18} + \frac{1}{18} = \frac{1}{9}$, identical to the upper bound.

cc. Let $p(-1) = \frac{1}{50}$, $p(+1) = \frac{1}{50}$, $p(0) = \frac{24}{25}$.

Section 3.4

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dd. $b(3;8,.35) = \binom{8}{3}(.35)^3(.65)^5 = .279$

ee. $b(5;8,.6) = \binom{8}{5}(.6)^5(.4)^3 = .279$

ff. $P(3 \le X \le 5) = b(3;7,.6) + b(4;7,.6) + b(5;7,.6) = .745$

gg. $P(1 \le X) = 1 - P(X = 0) = 1 - \binom{9}{0}(.1)^0(.9)^9 = 1 - (.9)^9 = .613$

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hh. $B(4;15,.3) = .515$

ii. $b(4;15,.3) = B(4;15,.3) - B(3;15,.3) = .219$

jj. $b(6;15,.7) = B(6;15,.7) - B(5;15,.7) = .012$

kk. $P(2 \le X \le 4) = B(4;15,.3) - B(1;15,.3) = .480$

ll. $P(2 \le X) = 1 - P(X \le 1) = 1 - B(1;15,.3) = .965$

mm. $P(X \le 1) = B(1;15,.7) = .000$

nn. $P(2 < X < 6) = P(2 < X \le 5) = B(5;15,.3) - B(2;15,.3) = .595$

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$X \sim \text{Bin}(6, .10)$

oo. $P(X = 1) = \binom{n}{x}(p)^x(1-p)^{n-x} = \binom{6}{1}(.1)^1(.9)^5 = .3543$

pp. $P(X \ge 2) = 1 - [P(X = 0) + P(X = 1)]$.

From **a** , we know P(X = 1) = .3543, and $P(X = 0) = \binom{6}{0}(.1)^0(.9)^6 = .5314$.

Hence $P(X \geq 2) = 1 - [.3543 + .5314] = .1143$

qq. Either 4 or 5 goblets must be selected

i) Select 4 goblets with zero defects: $P(X = 0) = \binom{4}{0}(.1)^0(.9)^4 = .6561$.

ii) Select 4 goblets, one of which has a defect, and the 5^{th} is good:

$$\left[\binom{4}{1}(.1)^1(.9)^3\right] \times .9 = .26244$$

So the desired probability is .6561 + .26244 = .91854

50 $X \sim Bin(25,.25)$

rr. $P(X \leq 6) = B(6;25,.25) = .561$

ss. $P(X = 6) = b(6;25,.25) = .183$

tt. $P(X \geq 6) = 1 - P(X \leq 5) = 1 - B(5;25,.25) = .622$

uu. $P(X > 6) = 1 - P(X \leq 6) = 1 - .561 = .439$

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Let S represent a telephone that is submitted for service while under warranty and must be replaced. Then p = P(S) = P(replaced | submitted)·P(submitted) = (.40)(.20) = .08. Thus X, the number among the company's 10 phones that must be replaced, has a binomial distribution with n = 10, p = .08, so $p(2) = P(X=2) = \binom{10}{2}(.08)^2(.92)^8 = .1478$

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X = the number of flashlights that work.
Let event B = {battery has acceptable voltage}.
Then P(flashlight works) = P(both batteries work) = P(B)P(B) = (.9)(.9) = .81 We must assume that the batteries' voltage levels are independent.
$X \sim Bin(10, .81)$. $P(X\geq 9) = P(X=9) + P(X=10) = .285 + .122 = .407$