Unit 2 Homework:

Chapter 3

69. $X \sim h(x; 6, 12, 7)$

a. $P(X=5) = \dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} = \dfrac{105}{924} = .114$

b. $P(X\le 4) = 1 - P(X\ge 5) = 1 - [P(X=5) + P(X=6)] = 1 - \left[\dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} + \dfrac{\binom{7}{6}}{\binom{12}{6}}\right] = 1 - \dfrac{105+7}{924} = 1 - .121 = .879$

c. $E(X) = \left(\dfrac{6\cdot 7}{12}\right) = 3.5$; $\sigma = \sqrt{\left(\frac{6}{11}\right)(6)\left(\frac{7}{12}\right)\left(\frac{5}{12}\right)} = \sqrt{.795} = .892$ $P(X > 3.5 + .892) = P(X > 4.392)$
$= P(X\ge 5) = .121$ (see part b)

d. We can approximate the hypergeometric distribution with the binomial if the population size and the number of successes are large: $h(x;15,40,400)$ approaches $b(x;15,.10)$. So $P(X\le 5) \approx B(5; 15, .10)$ from the binomial tables = .998

71.

e. Possible values of X are 5, 6, 7, 8, 9, 10. (In order to have less than 5 of the granite, there would have to be more than 10 of the basaltic).

$P(X = 5) = h(5; 15,10,20) = \dfrac{\binom{10}{5}\binom{10}{10}}{\binom{20}{15}} = .0163$.

Following the same pattern for the other values, we arrive at the pmf, in table form below.

x	5	6	7	8	9	10
p(x)	.0163	.1354	.3483	.3483	.1354	.0163

f. P(all 10 of one kind or the other) = P(X = 5) + P(X = 10) = .0163 + .0163 = .0326

g. $E(X) = n\cdot\dfrac{M}{N} = 15\cdot\dfrac{10}{20} = 7.5$; $V(X) = \left(\dfrac{5}{19}\right)(7.5)\left(1-\dfrac{10}{20}\right) = .9868$; $\sigma_x = .9934$

$\mu \pm \sigma = 7.5 \pm .9934 = (6.5066, 8.4934)$, so we want
$P(X = 7) + P(X = 8) = .3483 + .3483 = .6966$

73.

h. h(x; 10,10,20) (the successes here are the top 10 pairs, and a sample of 10 pairs is drawn from among the 20)

i. Let X = the number among the top 5 who play E-W. Then P(all of top 5 play the same direction) = P(X = 5) + P(X = 0)
= h(5;10,5,20) + h(5;10,5,20)

$$= \frac{\binom{15}{5}}{\binom{20}{10}} + \frac{\binom{15}{10}}{\binom{20}{10}} = .033$$

j. N = 2n; M = n; n = n
h(x;n,n,2n)

E(X) = $n \cdot \frac{n}{2n} = \frac{1}{2}n$;

V(X) = $\left(\frac{2n-n}{2n-1}\right) \cdot n \cdot \frac{n}{2n} \cdot \left(1 - \frac{n}{2n}\right) = \left(\frac{n}{2n-1}\right) \cdot \frac{n}{2} \cdot \left(1 - \frac{n}{2n}\right) = \left(\frac{n}{2n-1}\right) \cdot \frac{n}{2} \cdot \left(\frac{1}{2}\right)$

75.

k. With S = a female child and F = a male child, let X = the number of F's before the 2^{nd} S. Then P(X = x) = nb(x;2, .5)

l. P(exactly 4 children) = P(exactly 2 males)
= nb(2;2,.5) = (3)(.0625) = .188

m. P(at most 4 children) = P(X ≤ 2)

$$= \sum_{x=0}^{2} nb(x;2,.5) = .25+2(.25)(.5) + 3(.0625) = .688$$

n. E(X) = $\frac{(2)(.5)}{.5} = 2$, so the expected number of children = E(X + 2)
= E(X) + 2 = 4

79.

o. P(X ≤ 8) = F(8;5) = .932

p. P(X = 8) = F(8;5) - F(7;5) = .065

q. P(X ≥ 9) = 1 – P(X ≤ 8) = .068

r. P(5 ≤ X ≤ 8) = F(8;5) – F(4;5) = .492

s. P(5 < X < 8) = F(7;5) – F(5;5) = .867-.616=.251

81

t. $P(X \le 10) = F(10;20) = .011$

u. $P(X > 20) = 1 - F(20;20) = 1 - .559 = .441$

v. $P(10 \le X \le 20) = F(20;20) - F(9;20) = .559 - .005 = .554$
$P(10 < X < 20) = F(19;20) - F(10;20) = .470 - .011 = .459$

w. $E(X) = \lambda = 20$, $\sigma_X = \sqrt{\lambda} = 4.472$

$$\begin{aligned} P(\mu - 2\sigma < X < \mu + 2\sigma) &= P(20 - 8.944 < X < 20 + 8.944) \\ &= P(11.056 < X < 28.944) \\ &= P(X \le 28) - P(X \le 11) \\ &= F(28;20) - F(11;20) \\ &= .966 - .021 = .945 \end{aligned}$$

83

a. $p = \dfrac{1}{200}$; n = 1000; $\lambda = np = 5$

x. $P(5 \le X \le 8) = F(8;5) - F(4;5) = .492$

y. $P(X \ge 8) = 1 - P(X \le 7) = 1 - .867 = .133$

85

z. $\lambda = 8$ when t = 1, so $P(X = 6) = F(6;8) - F(5;8) = .313 - .191 = .122$,
$P(X \ge 6) = 1 - F(5;8) = .809$, and $P(X \ge 10) = 1 - F(9;8) = .283$

aa. t = 90 min = 1.5 hours, so $\lambda = 12$; thus the expected number of arrivals is 12 and the SD = $\sqrt{12} = 3.464$

bb. t = 2.5 hours implies that $\lambda = 20$; in this case, $P(X \ge 20) = 1 - F(19;20) = .530$ and $P(X \le 10) = F(10;20) = .011$.

Chapter 4

1.

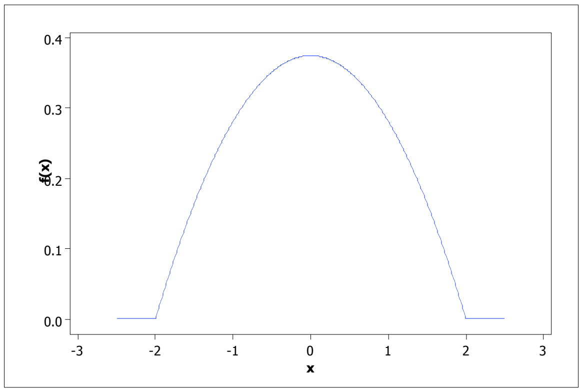
a. P(X ≤ 1) = $\int_{-\infty}^{1} f(x)dx = \int_0^1 \frac{1}{2}x dx = \frac{1}{4}x^2\Big]_0^1 = .25$

b. P(.5 ≤ X ≤ 1.5) = $\int_{.5}^{1.5} \frac{1}{2}x dx = \frac{1}{4}x^2\Big]_{.5}^{1.5} = .5$

c. P(X > 1.5) = $\int_{1.5}^{\infty} f(x)dx = \int_{1.5}^{2} \frac{1}{2}x dx = \frac{1}{4}x^2\Big]_{1.5}^{2} = \frac{7}{16} \approx .438$

3.

d.



e. P(X > 0) = $\int_0^2 .09375(4 - x^2)dx = .09375(4x - \frac{x^3}{3})\Big]_0^2 = .5$

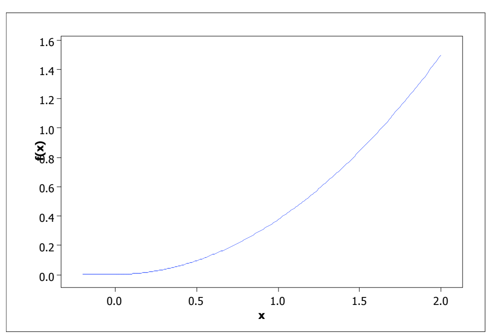
f. P(–1 < X < 1) = $\int_{-1}^{1} .09375(4 - x^2)dx = .6875$

g. P(x < –.5 OR x > .5) = 1 – P(–.5 ≤ X ≤ .5) = 1 – $\int_{-.5}^{.5} .09375(4 - x^2)dx$

= 1 – .3672 = .6328

5

h. 1 = $\int_{-\infty}^{\infty} f(x)dx = \int_0^2 kx^2 dx = k\left(\frac{x^3}{3}\right)\Big]_0^2 = k\left(\frac{8}{3}\right) \Rightarrow k = \frac{3}{8}$



i. P(0 ≤ X ≤ 1) = $\int_0^1 \frac{3}{8}x^2 dx = \frac{1}{8}x^3\Big]_0^1 = \frac{1}{8} = .125$

j. P(1 ≤ X ≤ 1.5) = $\int_1^{1.5} \frac{3}{8}x^2 dx = \frac{1}{8}x^3\Big]_1^{1.5} = \frac{1}{8}\left(\frac{3}{2}\right)^3 - \frac{1}{8}(1)^3 = \frac{19}{64} \approx .2969$

k. P(X ≥ 1.5) = 1 – $\int_0^{1.5} \frac{3}{8}x^2 dx = \frac{1}{8}x^3\Big]_0^{1.5} = 1 - \left[\frac{1}{8}\left(\frac{3}{2}\right)^3 - 0\right] = 1 - \frac{27}{64} = \frac{37}{64} \approx .5781$

11.

l. $P(X \le 1) = F(1) = \frac{1}{4} = .25$

m. $P(.5 \le X \le 1) = F(1) - F(.5) = \frac{3}{16} = .1875$

n. $P(X > .5) = 1 - P(X \le .5) = 1 - F(.5) = \frac{15}{16} = .9375$

o. $.5 = F(\tilde{\mu}) = \frac{\tilde{\mu}^2}{4} \Rightarrow \tilde{\mu}^2 = 2 \Rightarrow \tilde{\mu} = \sqrt{2} \approx 1.414$

p. $f(x) = F'(x) = \frac{x}{2}$ for $0 \le x < 2$, and = 0 otherwise

q. $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx = \int_0^2 x \cdot \frac{1}{2}xdx = \frac{1}{2}\int_0^2 x^2 dx = \frac{x^3}{6}\Big]_0^2 = \frac{8}{6} \approx 1.333$

r. $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^2 x^2 \frac{1}{2}xdx = \frac{1}{2}\int_0^2 x^3 dx = \frac{x^4}{8}\Big]_0^2 = 2,$

So $Var(X) = E(X^2) - [E(X)]^2 = 2 - \left(\frac{8}{6}\right)^2 = \frac{8}{36} \approx .222$, $\sigma_X \approx .471$

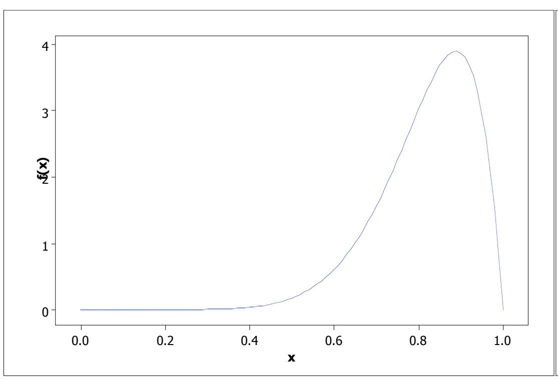
s. From **g**, $E(X^2) = 2$

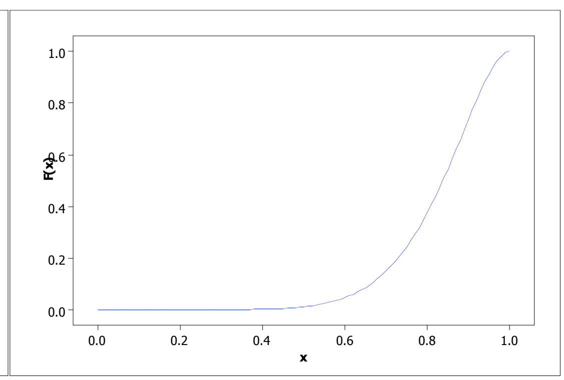
15

t. $F(x) = 0$ for $x \le 0$, $= 1$ for $x \ge 1$, and for $0 < x < 1$,

$$F(x) = \int_{-\infty}^{x} f(y)dy = \int_0^x 90y^8(1-y)dy = 90\int_0^x (y^8 - y^9)dy$$

$$90\left(\tfrac{1}{9}y^9 - \tfrac{1}{10}y^{10}\right)\Big]_0^x = 10x^9 - 9x^{10}$$





u. $F(.5) = 10(.5)^9 - 9(.5)^{10} \approx .0107$

v. $P(.25 \le X \le .5) = F(.5) - F(.25) \approx .0107 - [10(.25)^9 - 9(.25)^{10}]$
$\approx .0107 - .0000 \approx .0107$

w. The 75th percentile is the value of x for which $F(x) = .75$
$\Rightarrow .75 = 10(x)^9 - 9(x)^{10}$ $\Rightarrow x \approx .9036$

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x. E(X) = $\frac{1}{\lambda} = 1$

y. $\sigma = \frac{1}{\lambda} = 1$

z. P(X ≤ 4) = $1 - e^{-(1)(4)} = 1 - e^{-4} = .982$

aa. P(2 ≤ X ≤ 5) = $1 - e^{-(1)(5)} - \left[1 - e^{-(1)(2)}\right] = e^{-2} - e^{-5} = .129$

61

Mean = $\frac{1}{\lambda}$ = 25,000 implies λ = .00004

bb. P(X > 20,000) = 1 – P(X ≤ 20,000) = 1 – F(20,000; .00004) $= e^{-(.00004)(20,000)} = .449$

P(X ≤ 30,000) = F(30,000; .00004) $= 1 - e^{-1.2} = .699$

P(20,000 ≤ X ≤ 30,000) = .699 – .551 = .148

cc. $\sigma = \frac{1}{\lambda} = 25,000$, so P(X > μ + 2σ) = P(x > 75,000) =

1 – F(75,000;.00004) = .05.

Similarly, P(X > μ + 3σ) = P(x > 100,000) = .018

28

dd. P(0 ≤ Z ≤ 2.17) = Φ(2.17) – Φ(0) = .4850

ee. Φ(1) – Φ(0) = .3413

ff. Φ(0) – Φ(–2.50) = .4938

gg. Φ(2.50) – Φ(–2.50) = .9876

hh. Φ(1.37) = .9147

ii. P(–1.75 < Z) + [1 – P(Z < –1.75)] = 1 – Φ(–1.75) = .9599

jj. Φ(2) – Φ(–1.50) = .9104

kk. Φ(2.50) – Φ(1.37) = .0791

ll. 1 – Φ(1.50) = .0668

mm. P(|Z| ≤ 2.50) = P(–2.50 ≤ Z ≤ 2.50) = Φ(2.50) – Φ(–2.50) = .9876

31

nn. Area under Z curve above $z_{.0055}$ is .0055, which implies that $\Phi(z_{.0055}) = 1 - .0055 = .9945$, so $z_{.0055} = 2.54$

oo. $\Phi(z_{.09}) = .9100 \Rightarrow z = 1.34$ (since .9099 appears as the 1.34 entry).

pp. $\Phi(z_{.663})$ = area below $z_{.663} = .3370 \Rightarrow z_{.633} \approx -.42$

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qq. $P(X \le 18) = P\left(Z \le \frac{18-15}{1.25}\right) = P(Z \le 2.4) = \Phi(2.4) = .9452$

rr. $P(10 \le X \le 12) = P(-4.00 \le Z \le -2.40) \approx P(Z \le -2.40) = \Phi(-2.40) = .0082$

ss. $P(|X - 15| \le 1.5(1.25)) = P(|Z| \le 1.5) = P(-1.5 \le Z \le 1.5) = \Phi(1.5) - \Phi(-1.5) = .8664$

35

tt. $P(X \ge 10) = P(Z \ge .43) = 1 - \Phi(.43) = 1 - .6664 = .3336$.
$P(X > 10) = P(X \ge 10) = .3336$, since for any continuous distribution, $P(x = a) = 0$.

uu. $P(X > 20) = P(Z > 4) \approx 0$

vv. $P(5 \le X \le 10) = P(-1.36 \le Z \le .43) = \Phi(.43) - \Phi(-1.36) = .6664 - .0869 = .5795$

ww. $P(8.8 - c \le X \le 8.8 + c) = .98$, so $8.8 - c$ and $8.8 + c$ are at the 1st and the 99th percentile of the given distribution, respectively. The 1st percentile of the standard normal distribution has the value –2.33, so
$8.8 - c = \mu + (-2.33)\sigma = 8.8 - 2.33(2.8) \Rightarrow c = 2.33(2.8) = 6.524$.

xx. From a, $P(x > 10) = .3336$. Define event A as {diameter > 10}, then P(at least one A_i) = 1 – P(no A_i)
$= 1 - P(A')^4 = 1 - (1 - .3336)^4 = 1 - .1972 = .8028$

37

yy. $P(X = 105) = 0$, since the normal distribution is continuous;
$P(X < 105) = P(Z < 0.2) = P(Z \le 0.2) = \Phi(0.2) = .5793$;
$P(X \le 105) = .5793$ as well, since X is continuous

zz. No, the answer does not depend on μ or σ. For any normal rv, $P(|X-\mu| > \sigma) = P(|Z| > 1) = P(Z < -1 \text{ or } Z > 1) = 2\Phi(-1) = 2(.1587) = .3174$

aaa. From the table, $\Phi(z) = .1\% = .001 \rightarrow z \approx -3.09 \rightarrow x = 104 - 3.09(5) = 88.55$ mmol/L. The smallest .1% of chloride concentration values are those less than 88.55 mmol/L

47 The stated condition implies that 99% of the area under the normal curve with $\mu = 12$ and $\sigma = 3.5$ is to the left of $c - 1$, so $c - 1$ is the 99th percentile of the distribution. Thus $c - 1 = \mu + \sigma(2.33) = 20.155$, and $c = 21.155$.

14

bbb.

2	23	stem units: 1.0
3	2344567789	leaf units: .10
4	01356889	
5	00001114455666789	
6	0000122223344456667789999	
7	00012233455555668	
8	02233448	
9	012233335666788	
10	2344455688	
11	2335999	
12	37	
13	8	
14	36	
15	0035	
16		
17		
18	9	

ccc. A representative value could be the median, 7.0.

ddd. The data appear to be highly concentrated, except for a few values on the positive side.

eee. No, the data is skewed to the right, or positively skewed.

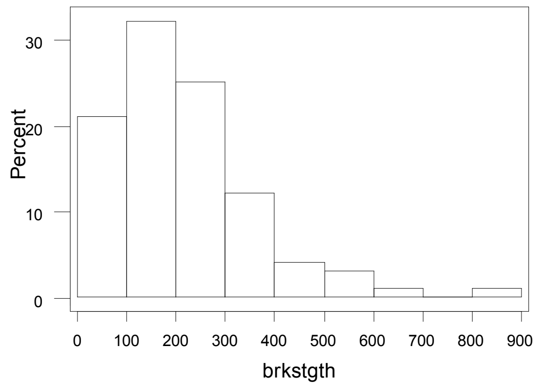
fff. The value 18.9 appears to be an outlier, being more than two stem units from the previous value.

15

Crunchy		Creamy
	2	2
644	3	69
77220	4	145
6320	5	3666
222	6	258
55	7	
0	8	

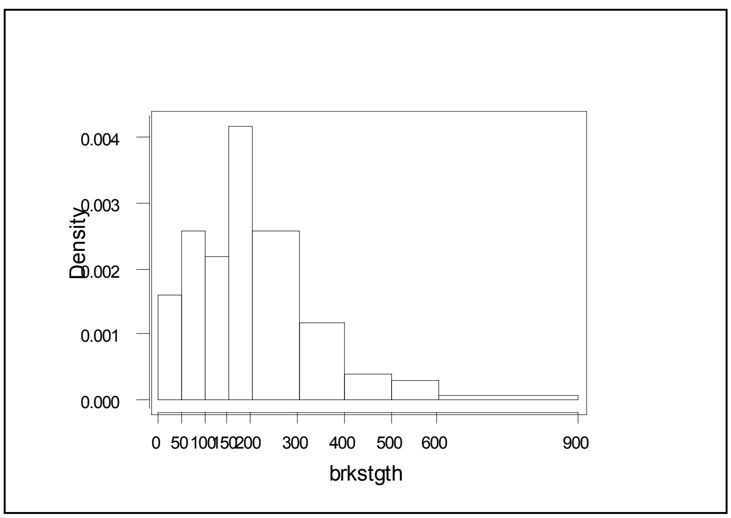
Both sets of scores are reasonably spread out. There appear to be no outliers. The three highest scores are for the crunchy peanut butter, the three lowest for the creamy peanut butter.

ggg.



The histogram is skewed right, with a majority of observations between 0 and 300 cycles. The class holding the most observations is between 100 and 200 cycles.

hhh.



c. [proportion ≥ 100] = 1 – [proportion < 100] = 1 – .21 = .79

33

iii. $\bar{x} = 192.57$, $\tilde{x} = 189$ .The mean is larger than the median, but they are still fairly close together.

jjj. Changing the one value, $\bar{x} = 189.71$, $\tilde{x} = 189$. The mean is lowered, the median stays the same.

kkk. $\bar{x}_{tr} = 191.0$. $\frac{1}{14} = .07$ or 7% trimmed from each tail.

lll. For n = 13, Σx = (119.7692) x 13 = 1,557
For n = 14, Σx = 1,557 + 159 = 1,716
$$\bar{x} = \frac{1716}{14} = 122.5714 \text{ or } 122.6$$

37 $\bar{x} = 12.01$, $\tilde{x} = 11.35$, $\bar{x}_{tr(10)} = 11.46$. The median or the trimmed mean would be good choices because of the outlier 21.9.

39

mmm. $\Sigma x_i = 16.475$ so $\bar{x} = \frac{16.475}{16} = 1.0297$; $\tilde{x} = \frac{(1.007 + 1.011)}{2} = 1.009$

nnn. 1.394 can be decreased until it reaches 1.011(the largest of the 2 middle values) – i.e. by 1.394 – 1.011 = .383, If it is decreased by more than .383, the median will change.

47 The sample median is $\tilde{x} = 114$, the sample mean is $\bar{x} = \frac{1}{n}\sum x_i = \frac{1}{10}(1{,}162) = \bar{x} = 116.2$.

The sample standard deviation, $s = \sqrt{\dfrac{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}}{n-1}} = \sqrt{\dfrac{140{,}992 - \frac{(1{,}162)^2}{10}}{9}} = 25.75$

On average, we would expect a fracture strength of 116.2. In general, the size of a typical deviation from the sample mean (116.2) is about 25.75. Some observations may deviate from 116.2 by more than this and some by less.

49

a. $\Sigma x = 2.75 + ... + 3.01 = 56.80$, $\Sigma x^2 = (2.75)^2 + ... + (3.01)^2 = 197.8040$

b. $s^2 = \frac{197.8040 - (56.80)^2 / 17}{16} = \frac{8.0252}{16} = .5016$, $s = .708$

51

c. $\Sigma x = 2563$ and $\Sigma x^2 = 368{,}501$, so

$$s^2 = \frac{[368{,}501 - (2563)^2 / 19]}{18} = 1264.766 \text{ and } s = 35.564$$

d. If y = time in minutes, then y = cx where $c = \frac{1}{60}$, so

$$s_y^2 = c^2 s_x^2 = \frac{1264.766}{3600} = .351 \text{ and } s_y = cs_x = \frac{35.564}{60} = .593$$

Chapter 6

1

ooo. We use the sample mean, $\bar{x}$ to estimate the population mean μ. $\hat{\mu} = \bar{x} = \frac{\Sigma x_i}{n} = \frac{219.80}{27} = 8.1407$

ppp. We use the sample median, $\tilde{x} = 7.7$ (the middle observation when arranged in ascending order).

qqq. We use the sample standard deviation, $s = \sqrt{s^2} = \sqrt{\frac{1860.94 - \frac{(219.8)^2}{27}}{26}} = 1.660$

rrr. With "success" = observation greater than 10, x = # of successes = 4, and $\hat{p} = \frac{x}{n} = \frac{4}{27} = .1481$

sss. We use the sample (std dev)/(mean), or $\frac{s}{\bar{x}} = \frac{1.660}{8.1407} = .2039$

3

a. We use the sample mean, $\bar{x} = 1.3481$

b. Because we assume normality, the mean = median, so we also use the sample mean $\bar{x} = 1.3481$. We could also easily use the sample median.

c. We use the 90th percentile of the sample: $\hat{\mu} + (1.28)\hat{\sigma} = \bar{x} + 1.28s = 1.3481 + (1.28)(.3385) = 1.7814$.

d. Since we can assume normality, $P(X < 1.5) \approx P\left(Z < \frac{1.5 - \bar{x}}{s}\right) = P\left(Z < \frac{1.5 - 1.3481}{.3385}\right) = P(Z < .45) = .6736$

e. The estimated standard error of $\bar{x} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{.3385}{\sqrt{16}} = .0846$

5 Let θ = the total audited value. Three potential estimators of θ are $\hat{\theta}_1 = N\bar{X}$, $\hat{\theta}_2 = T - N\bar{D}$, and $\hat{\theta}_3 = T \cdot \frac{\bar{X}}{\bar{Y}}$. From the data, $\bar{y} = 374.6$, $\bar{x} = 340.6$, and $\bar{d} = 34.0$. Knowing N = 5,000 and T = 1,761,300, the three corresponding estimates are $\hat{\theta}_1 = (5{,}000)(340.6) = 1{,}703{,}000$, $\hat{\theta}_2 = 1{,}761{,}300 - (5{,}000)(34.0) = 1{,}591{,}300$, and $\hat{\theta}_3 = 1{,}761{,}300\left(\frac{340.6}{374.6}\right) = 1{,}601{,}438.281$.

f. We wish to take the derivative of $\ln\left[\binom{n}{x}p^x(1-p)^{n-x}\right]$, set it equal to zero and solve for p.

$\frac{d}{dp}\left[\ln\binom{n}{x}+x\ln(p)+(n-x)\ln(1-p)\right]=\frac{x}{p}-\frac{n-x}{1-p}$; setting this equal to zero and solving for p yields $\hat{p}=\frac{x}{n}$.

For n = 20 and x = 3, $\hat{p}=\frac{3}{20}=.15$

g. $E(\hat{p})=E\left(\frac{X}{n}\right)=\frac{1}{n}E(X)=\frac{1}{n}(np)=p$; thus $\hat{p}$ is an unbiased estimator of p.

h. $(1-.15)^5=.4437$

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	$P(x_1)$	.20	.50	.30
$P(x_2)$	$x_2 \mid x_1$	25	40	65
.20	25	.04	.10	.06
.50	40	.10	.25	.15
.30	65	.06	.15	.09

a.

$\bar{x}$	25	32.5	40	45	52.5	65
$p(\bar{x})$	.04	.20	.25	.12	.30	.09

$$E(\bar{x}) = (25)(.04) + 32.5(.20) + \ldots + 65(.09) = 44.5 = \mu$$

b.

s^2	0	112.5	312.5	800
$P(s^2)$	.38	.20	.30	.12

$E(s^2) = 212.25 = \sigma^2$

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$\mu = 12$ cm $\qquad \sigma = .04$ cm

c. n = 16 $E(\overline{X}) = \mu = 12cm \quad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.04}{4} = .01cm$

d. n = 64 $E(\overline{X}) = \mu = 12cm \quad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.04}{8} = .005cm$

e. $\overline{X}$ is more likely to be within .01 cm of the mean (12 cm) with the second, larger, sample. This is due to the decreased variability of $\overline{X}$ with a larger sample size.

47. $\mu = 12$ cm $\qquad \sigma = .04$ cm

a. n = 16 $P(11.99 \le \overline{X} \le 12.01) = P\left(\frac{11.99 - 12}{.01} \le Z \le \frac{12.01 - 12}{.01}\right)$

$= P(-1 \le Z \le 1)$
$= \Phi(1) - \Phi(-1)$
$=.8413 - .1587$
$=.6826$

b. n = 25 P($\overline{X}$ > 12.01) = $P\left(Z > \frac{12.01-12}{.04/5}\right)$ = P(Z > 1.25)

= 1 - Φ(1.25)

= 1 - .8944

=.1056

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c. 11 P.M. – 6:50 P.M. = 250 minutes. With $T_0 = X_1 + \ldots + X_{40}$ = total grading time, $\mu_{T_0} = n\mu = (40)(6) = 240$ and

$\sigma_{T_0} = \sigma\sqrt{n} = 37.95,$ so P($T_0 \le 250$) ≈ $P\left(Z \le \frac{250-240}{37.95}\right) = P(Z \le .26) = .6026$

d. $P(T_0 > 260) = P\left(Z > \frac{260-240}{37.95}\right) = P(Z > .53) = .2981$

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μ = 50, σ = 1.2

e. n = 9

P($\overline{X}$ ≥ 51) = $P\left(Z \ge \frac{51-50}{1.2/\sqrt{9}}\right) = P(Z \ge 2.5) = 1 - .9938 = .0062$

f. n = 40

P($\overline{X}$ ≥ 51) = $P\left(Z \ge \frac{51-50}{1.2/\sqrt{40}}\right) = P(Z \ge 5.27) \approx 0$

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g. E($X_1 + X_2 + X_3$) = 180, V($X_1 + X_2 + X_3$) = 45, $\sigma_{x_1+x_2+x_3} = 6.708$

P($X_1 + X_2 + X_3 \le 200$) = $P\left(Z \le \frac{200-180}{6.708}\right) = P(Z \le 2.98) = .9986$

P($150 \le X_1 + X_2 + X_3 \le 200$) = $P(-4.47 \le Z \le 2.98) \approx .9986$

h. $\mu_{\overline{X}} = \mu = 60$, $\sigma_{\overline{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{\sqrt{15}}{\sqrt{3}} = 2.236$

$P(\overline{X} \ge 55) = P\left(Z \ge \frac{55-60}{2.236}\right) = P(Z \ge -2.236) = .9875$

$P(58 \le \overline{X} \le 62) = P(-.89 \le Z \le .89) = .6266$

i. E($X_1 - .5X_2 - .5X_3$) = 0;

V($X_1 - .5X_2 - .5X_3$) = $\sigma_1^2 + .25\sigma_2^2 + .25\sigma_3^2 = 22.5$, sd = 4.7434

P($-10 \le X_1 - .5X_2 - .5X_3 \le 5$) = $P\left(\frac{-10-0}{4.7434} \le Z \le \frac{5-0}{4.7434}\right)$

$= P(-2.11 \le Z \le 1.05)$ = .8531 - .0174 = .8357

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μ = 5.00, σ = .2

j. $E(\overline{X}-\overline{Y})=0$; $V(\overline{X}-\overline{Y})=\dfrac{\sigma^2}{25}+\dfrac{\sigma^2}{25}=.0032$, $\sigma_{\overline{X}-\overline{Y}}=.0566$

$\Rightarrow P\left(-.1 \le \overline{X}-\overline{Y} \le .1\right)=P\left(-1.77 \le Z \le 1.77\right)=.9232$

k. $V(\overline{X}-\overline{Y})=\dfrac{\sigma^2}{36}+\dfrac{\sigma^2}{36}=.0022222$, $\sigma_{\overline{X}-\overline{Y}}=.0471$

$\Rightarrow P\left(-.1 \le \overline{X}-\overline{Y} \le .1\right)\approx P\left(-2.12 \le Z \le 2.12\right)=.9660$ (by the CLT)