Chapter 7

1.

a. $z_{\alpha/2} = 2.81$ implies that $\alpha/2 = 1 - \Phi(2.81) = .0025$, so $\alpha = .005$ and the confidence level is $100(1-\alpha)\% = 99.5\%$.

b. $z_{\alpha/2} = 1.44$ for $\alpha = 2[1 - \Phi(1.44)] = .15$, and $100(1-\alpha)\% = 85\%$.

c. 99.7% implies that $\alpha = .003$, $\alpha/2 = .0015$, and $z_{.0015} = 2.96$. (Look for cumulative area .9985 in the main body of table A.3, the Z table.)

d. 75% implies $\alpha = .25$, $\alpha/2 = .125$, and $z_{.125} = 1.15$.

3

e. A 90% confidence interval will be narrower. The z critical value for a 90% confidence level is 1.645, smaller than the z of 1.96 for the 95% confidence level, thus producing a narrower interval.

f. Not a correct statement. Once and interval has been created from a sample, the mean μ is either enclosed by it, or not. We have 95% confidence in the general procedure, under repeated and independent sampling.

g. Not a correct statement. The interval is an estimate for the population mean, not a boundary for population values.

h. Not a correct statement. In theory, if the process were repeated an infinite number of times, 95% of the intervals would contain the population mean μ. We *expect* 95 out of 100 intervals will contain μ, but we don't know this to be true.

5

i. $4.85 \pm \frac{(1.96)(.75)}{\sqrt{20}} = 4.85 \pm .33 =$ (4.52, 5.18).

j. $z_{\alpha/2} = z_{.02/2} = z_{.01} = 2.33$, so the interval is $4.56 \pm \frac{(2.33)(.75)}{\sqrt{16}} =$ (4.12, 5.00).

k. $n = \left[\frac{2(1.96)(.75)}{.40}\right]^2 = 54.02$, so n = 55.

l. w = 2(.2) = .4, so $n = \left[\frac{2(2.58)(.75)}{.4}\right]^2 = 93.61$, so n = 94.

13

m. $\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} = 654.16 \pm 1.96\frac{164.43}{\sqrt{50}}$ = (608.58, 699.74). We are 95% confident that the true average CO_2 level in this population of homes with gas cooking appliances is between 608.58ppm and 699.74ppm

n. $w = 50 = \frac{2(1.96)(175)}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{2(1.96)(175)}{50} = 13.72 \rightarrow n = (13.72)^2 = 188.24 \uparrow 189$

17

$\bar{x} - z_{.01}\frac{s}{\sqrt{n}} = 135.39 - 2.33\frac{4.59}{\sqrt{153}} = 135.39 - .865 = 134.53$. We are 99% confident that the true average ultimate tensile strength is greater than 134.53.

19

$\hat{p} = \frac{201}{356} = .5646$; We calculate a 95% confidence interval for the proportion of all dies that pass the probe:

$$\frac{.5646 + \frac{(1.96)^2}{2(356)} \pm 1.96\sqrt{\frac{(.5646)(.4354)}{356} + \frac{(1.96)^2}{4(356)^2}}}{1 + \frac{(1.96)^2}{356}} = \frac{.5700 \pm .0518}{1.01079} = (.513, .615)$$

23

o. $\hat{p} = \frac{24}{37} = .6486$; The 99% confidence interval for p is

$$\frac{.6486 + \frac{(2.58)^2}{2(37)} \pm 2.58\sqrt{\frac{(.6486)(.3514)}{37} + \frac{(2.58)^2}{4(37)^2}}}{1 + \frac{(2.58)^2}{37}} = \frac{.7386 \pm .2216}{1.1799} = (.438, .814)$$

p. $n = \frac{2(2.58)^2(.25) - (2.58)^2(.01) \pm \sqrt{4(2.58)^4(.25)(.25 - .01) + .01(2.58)^4}}{.01} = \frac{3.261636 \pm 3.3282}{.01} \approx 659$

29

q. $t_{.025,10} = 2.228$

r. $t_{.025,20} = 2.086$

s. $t_{.005,20} = 2.845$

t. $t_{.005,50} = 2.678$

u. $t_{.01,25} = 2.485$

v. $-t_{.025,5} = -2.571$

30

w. $t_{.025,10} = 2.228$

x. $t_{.025,15} = 2.131$

y. $t_{.005,15} = 2.947$

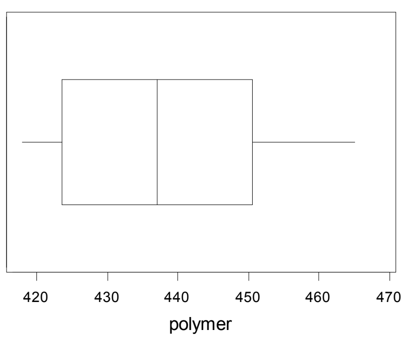
z. $t_{.005,4} = 4.604$

aa. $t_{.01,24} = 2.492$

bb. $t_{.005,37} \approx 2.712$

33

cc. The boxplot indicates a very slight positive skew, with no outliers. The data appears to center near 438.



dd. Based on a normal probability plot, it is reasonable to assume the sample observations came from a normal distribution.

ee. With d.f. = n – 1 = 16, the critical value for a 95% C.I. is $t_{.025,16} = 2.120$, and the interval is $438.29 \pm (2.120)\left(\frac{15.14}{\sqrt{17}}\right) = 438.29 \pm 7.785 = (430.51, 446.08)$. Since 440 is within the interval, 440 is a plausible value for the true mean. 450, however, is not, since it lies outside the interval.

35

$n = 15$, $\bar{x} = 25.0$, $s = 3.5$; $t_{.025,15} = 2.131$

ff. A 95% C.I. for the mean: $25.0 \pm 2.131\frac{3.5}{\sqrt{15}} = (23.1, 26.9)$

gg. A 95% Prediction Interval: $25.0 \pm 2.131(3.5)\sqrt{1+\frac{1}{15}}$ = (17.3, 32.7). The prediction interval is about 4 times wider than the confidence interval.

37

hh. A 95% C.I. : $.9255 \pm 2.093(.0181) = .9255 \pm .0379 \Rightarrow (.8876,.9634)$

ii. A 95% P.I. : $.9255 \pm 2.093(.0809)\sqrt{1+\frac{1}{20}} = .9255 \pm .1735 \Rightarrow (.7520,1.0990)$

jj. A tolerance interval is requested, with k = 99, confidence level 95%, and n = 20. The tolerance critical value, from Table A.6, is 3.615. The interval is $.9255 \pm 3.615(.0809) \Rightarrow (.6330,1.2180)$.

Chapter 8

1

kk. Yes. It is an assertion about the value of a parameter.

ll. No. The sample median $\tilde{X}$ is not a parameter.

mm. No. The sample standard deviation s is not a parameter.

nn. Yes. The assertion is that the standard deviation of population #2 exceeds that of population #1

oo. No. $\overline{X}$ and $\overline{Y}$ are statistics rather than parameters, so cannot appear in a hypothesis.

pp. Yes. H is an assertion about the value of a parameter.

3

In this formulation, H_0 states the welds do not conform to specification. This assertion will not be rejected unless there is strong evidence to the contrary. Thus the burden of proof is on those who wish to assert that the specification is satisfied. Using H_a: $\mu < 100$ results in the welds being believed in conformance unless proved otherwise, so the burden of proof is on the non-conformance claim.

5

Let σ denote the population standard deviation. The appropriate hypotheses are $H_o: \sigma = .05$ v. $H_a: \sigma < .05$. With this formulation, the burden of proof is on the data to show that the requirement has been met (the sheaths will not be used unless H_0 can be rejected in favor of H_a. Type I error: Conclude that the standard deviation is < .05 mm when it is really equal to .05 mm. Type II error: Conclude that the standard deviation is .05 mm when it is really < .05.

7

A type I error here involves saying that the plant is not in compliance when in fact it is. A type II error occurs when we conclude that the plant is in compliance when in fact it isn't. Reasonable people may disagree as to which of the two errors is more serious. If in your judgement it is the type II error, then the reformulation $H_o: \mu = 150$ v. $H_a: \mu < 150$ makes the type I error more serious.

17

qq. $z = \dfrac{30{,}960 - 30{,}000}{1500/\sqrt{16}} = 2.56 > 2.33$ so reject H_0.

rr. $\beta(30{,}500): \Phi\left(2.33 + \dfrac{30{,}000 - 30{,}500}{1500/\sqrt{16}}\right) = \Phi(1.00) = .8413$

ss. $\beta(30{,}500) = .05: n = \left[\dfrac{1500(2.33 + 1.645)}{30{,}000 - 30{,}500}\right]^2 = 142.2$, so use n = 143

tt. $\alpha = 1 - \Phi(2.56) = .0052$

19

uu. Reject H_0 if either $z \geq 2.58$ or $z \leq -2.58$; $\dfrac{\sigma}{\sqrt{n}} = 0.3$, so $z = \dfrac{94.32-95}{0.3} = -2.27$. Since –2.27 is not < -2.58, don't reject H_0.

vv. $\beta(94) = \Phi\left(2.58 + \dfrac{1}{0.3}\right) - \Phi\left(-2.58 + \dfrac{1}{0.3}\right) = \Phi(5.91) - \Phi(.75) = .2266$

ww. $n = \left[\dfrac{1.20(2.58+1.28)}{95-94}\right]^2 = 21.46$, so use n = 22.

21

With H_0: $\mu = .5$, and H_a: $\mu \neq .5$ we reject H_0 if $t > t_{\alpha/2,n-1}$ or $t < -t_{\alpha/2,n-1}$

xx. $1.6 < t_{.025,12} = 2.179$, so don't reject H_0

yy. $-1.6 > -t_{.025,12} = -2.179$, so don't reject H_0

zz. $-2.6 > -t_{.005,24} = -2.797$, so don't reject H_0

aaa. –3.9 < the negative of all t values in the df = 24 row, so we reject H_0 in favor of H_a.

23

H_0: $\mu = 360$ v.. H_a: $\mu > 360$; $t = \dfrac{\bar{x}-360}{s/\sqrt{n}}$; reject H_0 if $t > t_{.05,25} = 1.708$; $t = \dfrac{370.69-360}{24.36/\sqrt{26}} = 2.24 > 1.708$.
Thus H_0 should be rejected. There appears to be a contradiction of the prior belief.

25

bbb. H_0: $\mu = 5.5$ v.. H_a: $\mu \neq 5.5$; for a level .01 test, (not specified in the problem description), reject H_0 if either $z \geq 2.58$ or $z \leq -2.58$. Since $z = \dfrac{5.25-5.5}{.075} = -3.33 \leq -2.58$, reject H_0.

ccc. $1-\beta(5.6) = 1 - \Phi\left(2.58 + \dfrac{(-.1)}{.075}\right) + \Phi\left(-2.58 + \dfrac{(-.1)}{.075}\right) = 1 - \Phi(1.25) + \Phi(-3.91) = .105$

ddd. $n = \left[\dfrac{.3(2.58+2.33)}{-.1}\right]^2 = 216.97$, so use n = 217.

35

1 Parameter of interest: p = true proportion of cars in this particular county passing emissions testing on the first try.

2 H_0: p = .70

3 H_a: p ≠ .70

4 $z = \dfrac{\hat{p}-p_o}{\sqrt{p_o(1-p_o)/n}} = \dfrac{\hat{p}-.70}{\sqrt{.70(.30)/n}}$

5 either z ≥ 1.96 or z ≤ -1.96

6 $$z = \frac{124/200 - .70}{\sqrt{.70(.30)/200}} = -2.469$$

7 Reject H_0. The data indicates that the proportion of cars passing the first time on emission testing or this county differs from the proportion of cars passing statewide.

37

1 p = true proportion of all donors with type A blood

2 H_0: p = .40

3 H_a: p ≠ .40

4 $$z = \frac{\hat{p} - p_o}{\sqrt{p_o(1 - p_o)/n}} = \frac{\hat{p} - .40}{\sqrt{.40(.60)/n}}$$

5 Reject H_0 if $z \geq 2.58$ or $z \leq -2.58$

6 $$z = \frac{82/150 - .40}{\sqrt{.40(.60)/150}} = \frac{.147}{.04} = 3.667$$

7 Reject H_0. The data does suggest that the percentage of all donors with type A blood differs from 40%. (at the .01 significance level). Since the z critical value for a significance level of .05 is less than that of .01, the conclusion would not change.

39

eee. We wish to test H_0: p = .02 v. H_a: p < .02; only if H_0 can be rejected will the inventory be postponed. The lower-tailed test rejects H_0 if $z \leq -1.645$. With $\hat{p} = \frac{15}{1000} = .015$, z = -1.01, which is not ≤ -1.645. Thus, H_0 cannot be rejected, so the inventory should be carried out.

fff. $$\beta(.01) = 1 - \Phi\left[\frac{.02 - .01 - 1.645\sqrt{.02(.98)/1000}}{\sqrt{.01(.99)/1000}}\right] = 1 - \Phi(0.86) = .1949$$

ggg. $\beta(.05) = 1 - \Phi\left[\frac{.02 - .05 - 1.645\sqrt{.02(.98)/1000}}{\sqrt{.05(.95)/1000}}\right] = 1 - \Phi(-5.41) \approx 1$, so the chance the inventory will be *postoned* is P(reject H_0 when p = .05) = 1 – β(.05) = 0. It is highly unlikely that H_0 will be rejected, and the inventory will almost surely be carried out.

45

Using α = .05, H_0 should be rejected whenever P-value < .05.

hhh. P-value = .001 < .05, so reject H_0

iii. .021 < .05, so reject H_0.

jjj. .078 is not < .05, so don't reject H_0.

kkk. .047 < .05, so reject H_0 (a close call).

lll. .148 > .05, so H_0 can't be rejected at level .05.

47

In each case the p-value =P(Z > z) = $1 - \Phi(z)$

mmm. .0778

nnn. .1841

ooo..0250

ppp. .0066

qqq. .5438

51

By guessing alone, the taster has a 1/3 chance of selecting the "different" wine. Hence, we wish to test H_0: p= 1/3 v. H_a: p > 1/3. With $\hat{p} = \frac{346}{855} = .4047$, our test statistic is $z = \frac{.4047 - .3333}{\sqrt{.3333(.6667)/855}} = 4.43$, and the corresponding *P*-value is P(Z ≥ 4.43) ≈ 0. Hence, we strongly reject the null hypothesis at any reasonable significance level and conclude that the population of wine tasters have the ability to distinguish the "different" wine out of three more than 1/3 of the time.

53

rrr. For testing H_0: p = .2 v. H_a: p > .2, an upper-tailed test is appropriate. The computed Z is z = .97, so p-value = $1 - \Phi(.97) = .166$. Because the p-value is rather large, H_0 would not be rejected at any reasonable α (it can't be rejected for any α < .166), so no modification appears necessary.

sss. With p = .5, $1 - \beta(.5) = 1 - \Phi[(-.3 + 2.33(.0516))/.0645] = 1 - \Phi(-2.79) = .9974$

Chapter 9

1

ttt. $E(\overline{X}-\overline{Y}) = E(\overline{X}) - E(\overline{Y}) = 4.1 - 4.5 = -.4$, irrespective of sample sizes.

uuu. $V(\overline{X}-\overline{Y}) = V(\overline{X}) + V(\overline{Y}) = \dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n} = \dfrac{(1.8)^2}{100} + \dfrac{(2.0)^2}{100} = .0724$, and the s.d. of $\overline{X}-\overline{Y} = \sqrt{.0724} = .2691$.

vvv. A normal curve with mean and s.d. as given in **a** and **b** (because m = n = 100, the CLT implies that both $\overline{X}$ and $\overline{Y}$ have approximately normal distributions, so $\overline{X}-\overline{Y}$ does also). The shape is not necessarily that of a normal curve when m = n = 10, because the CLT cannot be invoked. So if the two lifetime population distributions are not normal, the distribution of $\overline{X}-\overline{Y}$ will typically be quite complicated.

21

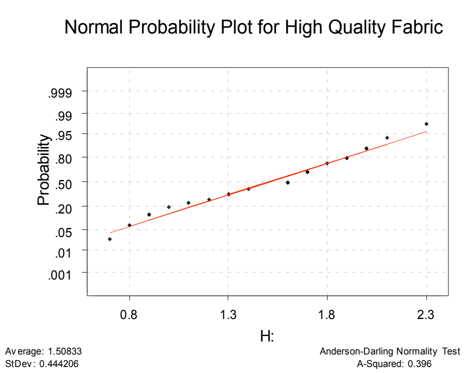
Let μ_1 = the true average gap detection threshold for normal subjects, and μ_2 = the corresponding value for CTS subjects. The relevant hypotheses are H_0: $\mu_1 - \mu_2 = 0$ vs. H_a: $\mu_1 - \mu_2 < 0$, and the test statistic

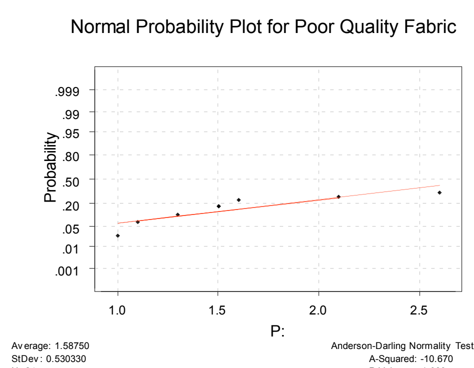
$$t = \frac{1.71 - 2.53}{\sqrt{.0351125 + .07569}} = \frac{-.82}{.3329} = -2.46.$$ Using d.f. $$\nu = \frac{(.0351125 + .07569)^2}{\frac{(.0351125)^2}{7} + \frac{(.07569)^2}{9}} = 15.1,$$ or 15, the

rejection region is $t \le -t_{.01,15} = -2.602$. Since –2.46 is not ≤ -2.602, we fail to reject H_0. We have insufficient evidence to claim that the true average gap detection threshold for CTS subjects exceeds that for normal subjects.

23

a.

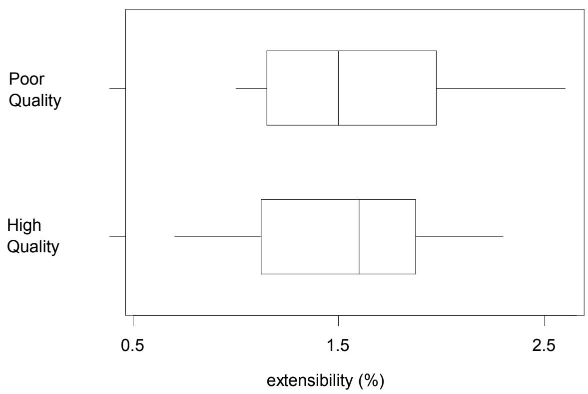




Using Minitab to generate normal probability plots, we see that both plots illustrate sufficient linearity. Therefore, it is plausible that both samples have been selected from normal population distributions.

b.

Comparative Box Plot for High Quality and Poor Quality Fabric



The comparative boxplot does not suggest a difference between average extensibility for the two types of fabrics.

c. We test $H_0 : \mu_1 - \mu_2 = 0$ vs. $H_a : \mu_1 - \mu_2 \neq 0$. With degrees of freedom $\nu = \frac{(.0433265)^2}{.00017906} = 10.5$, which we round down to 10, and using significance level .05 (not specified in the problem), we reject H$_0$ if $|t| \geq t_{.025,10} = 2.228$. The test statistic is $t = \frac{-.08}{\sqrt{(.0433265)}} = -.38$, which is not ≥ 2.228 in absolute value, so we cannot reject H$_0$. There is insufficient evidence to claim that the true average extensibility differs for the two types of fabrics.

25

We calculate the degrees of freedom $\nu = \frac{\left(\frac{5.5^2}{28} + \frac{7.8^2}{31}\right)^2}{\frac{\left(\frac{5.5^2}{28}\right)^2}{27} + \frac{\left(\frac{7.8^2}{31}\right)^2}{30}} = 53.95$, or about 54 (normally we would round down to 53, but this number is very close to 54 – of course for this large number of df, using either 53 or 54 won't make much difference in the critical t value) so the desired confidence interval is $(91.5 - 88.3) \pm 1.68\sqrt{\frac{5.5^2}{28} + \frac{7.8^2}{31}}$ $= 3.2 \pm 2.931 = (.269, 6.131)$. Because 0 does not lie inside this interval, we can be reasonably certain that the true difference $\mu_1 - \mu_2$ is not 0 and, therefore, that the two population means are not equal. For a 95% interval, the t value increases to about 2.01 or so, which results in the interval 3.2 ± 3.506. Since this interval does contain 0, we can no longer conclude that the means are different if we use a 95% confidence interval.

27

The approximate degrees of freedom for this estimate are $\nu = \frac{\left(\frac{11.3^2}{6} + \frac{8.3^2}{8}\right)^2}{\frac{\left(\frac{11.3^2}{6}\right)^2}{5} + \frac{\left(\frac{8.3^2}{8}\right)^2}{7}} = \frac{893.59}{101.175} = 8.83$, which we round down to 8, so $t_{.025,8} = 2.306$ and the desired interval is $(40.3 - 21.4) \pm 2.306\sqrt{\frac{11.3^2}{6} + \frac{8.3^2}{8}} = 18.9 \pm 2.306(5.4674)$ $= 18.9 \pm 12.607 = (6.3, 31.5)$. Because 0 is not contained in this interval, there is strong evidence that $\mu_1 - \mu_2$ is not 0;

i.e., we can conclude that the population means are not equal. Calculating a confidence interval for $\mu_2 - \mu_1$ would change only the order of subtraction of the sample means, but the standard error calculation would give the same result as before. Therefore, the 95% interval estimate of $\mu_2 - \mu_1$ would be (-31.5, -6.3), just the negatives of the endpoints of the original interval. Since 0 is not in this interval, we reach exactly the same conclusion as before; the population means are not equal.

39

d. A normal probability plot shows that the data could easily follow a normal distribution.

e. We test $H_0 : \mu_d = 0$ vs. $H_a : \mu_d \neq 0$, with test statistic $t = \frac{\bar{d} - 0}{s_D / \sqrt{n}} = \frac{167.2 - 0}{228 / \sqrt{14}} = 2.74 \approx 2.7$. The two-tailed p-value is 2[P(t > 2.7)] = 2[.009] = .018. Since .018 < .05, we can reject H_0. There is strong evidence to support the claim that the true average difference between intake values measured by the two methods is not 0. There is a difference between them.

41

We test $H_0 : \mu_d = 5$ vs. $H_a : \mu_d > 5$. With $\bar{d} = 7.600$, and $s_d = 4.178$, $t = \frac{7.600 - 5}{4.178 / \sqrt{9}} = \frac{2.6}{1.39} = 1.87 \approx 1.9$. With degrees of freedom n – 1 = 8, the corresponding p-value is P(t > 1.9) = .047. We would reject H_0 at any alpha level greater than .047. So, at the typical significance level of .05, we would reject H_0, and conclude that the data indicates that the higher level of illumination yields a decrease of more than 5 seconds in true average task completion time.