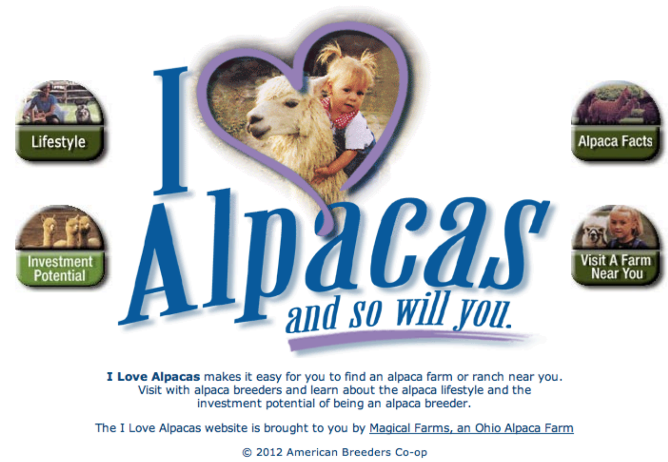
Activity #10: Quadratic Functions

(student learning outcomes listed in syllabus)

1) Your part-time job just isn't cutting it. You need to make some money, so you decide to start your own business. As you're brainstorming ideas, you run across the website ilovealpacas.com. You click the "investment potential" link and read:

> *Alpacas have been called the world's finest livestock investment. It is difficult to compare alpacas with other investments as pure investments. How much is peace of mind worth? Unlike the stock market, alpacas are depreciable over five years, giving the investor an immediate investment return in tax savings while the herd is growing.*



The website also explains the tax advantages and potential profits you could make from raising alpacas.

As you walk to class that afternoon, you happen to run across 6 wild alpacas – 3 males and 3 females. Serendipitously, you also find out that you won a plot of land and 1,200 yards of fencing in a contest at the local home improvement store.*

Since you're just beginning your career as an alpaca farmer, you don't want any additional alpacas at the moment. You decide to use the fence to create two adjacent areas for your alpacas (one for the males; one for the females).

What is the largest possible area you can get from your 1,200 yards of fence? While you only have the skills to build rectangular areas, you do not have to build equal areas for the male and female alpacas.

a) Draw and label a picture of this situation. Whoever can draw the most realistic alpaca in 1 minute wins.

b) Use the given information to write out the formulas of interest.

c) Use the substitution method to find the formula for the area in terms of the length of one of the sides.

Source: http://www.ilovealpacas.com/investment.shtml

*You try to think of some way a person could get alpacas, land, and a fence when they don't have much money.

A quadratic function of degree *n* has the form: $f(x) = ax^2 + bx + c$.

d) Graph this quadratic function and sketch it below. Using your calculator, find the maximum area that can be created from 1,200 yards of fence. What are the domain and range of this function? Over what intervals is it increasing/ decreasing? Is it concave up or concave down?

e) What are the dimensions of the maximum area you can enclose for your alpacas?

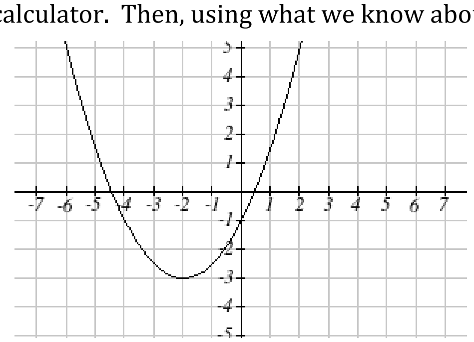
f) Bonus question: Suppose we need just one large area (we don't need to separate males and females). What is the maximum area you could enclose with 1200 yards of fence? Suppose you could make one large circle from the fencing, what area would be enclosed by the circle

A quadratic function with a vertex located at the point (h, k) can also be written as:

$$f(x) = a(x-h)^2 + k$$

where x = h represents the axis of symmetry, and
a represents the concavity (a>0 is concave up; a<0 is concave down)

2) Graph the base quadratic function, $y = x^2$, on your calculator. Then, using what we know about transformations, find the equation for the following quadratic function.



3) Identify the vertex, concavity, and axis of symmetry for the following quadratic functions. Sketch the graph of each function.

$f(x) = -2(x+4)^2 - 7$

$g(x) = 3(x-2)^2$

$h(x) = x^2 - 6x + 8$

4) To get that last one in vertex form, we can *complete the square:*

Completing the square:

Goal: To convert $f(x) = ax^2 + bx + c$ into $f(x) = a(x-h)^2 + k$

1. Divide everything by *a*

2. Add $\frac{b^2}{4a^2}$ after $\frac{b}{a}x$. Subtract $\frac{b^2}{4a^2}$ after $\frac{c}{a}$.

3. Factor: $a\left[\left(x+\frac{b}{2a}\right)^2 + c - \left(\frac{b^2}{4a^2}\right)\right]$

Complete the square for the following quadratic expressions:

$x^2 - 6x + 8$

$-4x^2 - 12x - 8$

$ax^2 + bx + c = 0$

5) Solve the following quadratic equations graphically and by using the quadratic formula:

$$-4x^2 - 12x - 8 = 0$$

$$2x^2 + 4x - 4 = 0$$

$$x^2 - 6x + 9 = 0$$

$$2x^2 - x + 4 = 0$$

6) What are the maximum and minimum number of real zeros a quadratic function can have? Sketch graphs of each case.

7) When we deal with quadratic functions, we're often interested in identifying the zeros and the vertex. We just practiced finding zeros, so let's practice finding vertices.

As we saw in question #4, when a quadratic function is written in vertex form, it's easy to identify the vertex. When a quadratic function is written in standard form, it's more difficult.

Let's set the vertex form equal to the standard form and see if we can derive an expression we can use to calculate the vertex of any quadratic function.

$$a(x-h)^2 + k = ax^2 + bx + c$$

To get started, let's expand the vertex form:

$$a(x-h)^2 + k =$$

Now, set this expanded form equal to the standard form of a quadratic function. Solve for the 1st-degree (linear) term. This is the expression you can use to calculate the vertex of a quadratic function.

8) Find the vertex of the following quadratic functions:

$$2x^2 - 6x + 7$$

$$-2x^2 + 80x$$

9) Suppose we have a quadratic function with a vertex at (2, 3). If I tell you the point (7, 10) is also on the graph of this function, find the function.

10) A local newspaper has 84,000 subscribers at a quarterly charge of \$30. Market research suggests that if they raise the price to \$32, they would lose 5,000 subscribers. If we assume subscriptions are linearly related to price, what price should the newspaper charge for a quarterly subscription to maximize their revenue?

First, develop a linear model to express the number of subscribers, S, as a function of the amount charged, C.

We know that the revenue (R) generated by subscriptions is modeled by R = CS. Substitute the linear function you just derived into the function for revenue. Write this quadratic function in standard form.

Find the vertex of this quadratic function and explain what it represents.

11) Ignoring air resistance, the height (H) of an objected projected upward from an initial height of h_0 with initial velocity V_0 is modeled by:

$$H(t) = -16t^2 + v_0 t + h_0$$

Suppose Justin Verlander throws a baseball directly upward from an initial height of 6 feet with an initial velocity of 150 feet per second.

a) Give the function that describes the height of the baseball as a function of time.
b) Sketch a graph of the function and identify the domain and range.
c) Determine the height of the ball after 4 seconds.
d) At what time does the ball reach its maximum height?
e) How high is the ball at its maximum?
f) How many seconds does it take for the ball to hit the ground?

12) Let's finish this activity by fitting a quadratic function to a set of data. We'll once again use our calculators to find the best-fitting quadratic function (QuadReg).

The Gateway Arch in St. Louis stands 630 feet tall. The distance between its legs is also 630 feet. If we superimpose a grid on the picture of the arch, we could find the following points:

X	Height
-325	0
-300	100
-250	330
-200	500
-150	570
-100	610
0	630
100	610
150	570
200	500
250	330
300	100
325	0

Enter this data into your calculator. Find the quadratic function that best fits this data. Write out the formula and comment on how well the function fits the data.

Then, use your calculator to fit a 4th-degree polynomial (QuartReg) to the data. Which function better fits the data?

Note: The true shape of the arch is a curve known as a hyperbolic cosine.