Activity #13: Periodic Functions (student learning outcomes listed in syllabus)

1) According to the New York City mayor's office, the world's largest Ferris Wheel is being planned for Staten Island. The Ferris Wheel, four times the size of the one at Navy Pier, will stand 625 feet tall and carry 1,440 passengers. A single ride will last 38 minutes.

Suppose you ride this Ferris Wheel and are interested in your height as a function of time (minutes since you boarded the Ferris Wheel). Complete the following table:

A rendering of the New York Wheel on Staten Island.

Source: http://en.wikipedia.org/wiki/New_York_Wheel#New_York_Wheel
http://www.travelweekly.com/uploadedImages/All_TW_Destinations/USA_Canada/2012/NewYorkWheel-render.jpg

t minutes	0	9.5	19	28.5	38	47.5	57	66.5	76
h(t) Height	0								0

2) Display this function on a graph. Should you connect the points with straight lines? Explain.

3) This is an example of a *periodic function*. The shortest distance on the graph in which the function completes one full cycle is called its *period.* What is the period for our function? If we took the graph of our function and shifted it to the left or right by its period, what would happen to the graph?

4) The *amplitude* of a function is one half of the distance between its minimum and maximum values. What is the amplitude of our function?

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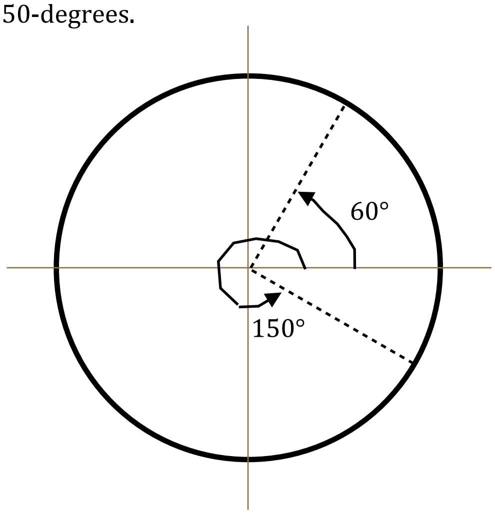
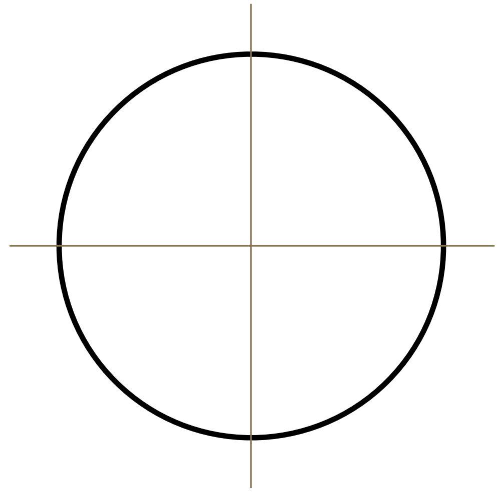
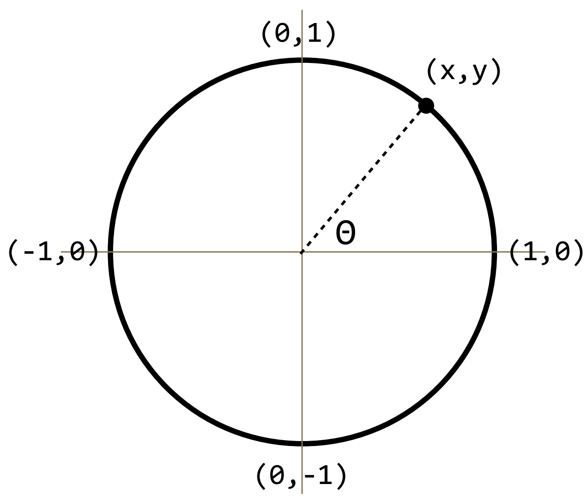
5) Let's try to construct a formula for our function. To do this, we'll first represent positions on the Ferris Wheel using angles. By convention, angles are measured counterclockwise from the x-axis. For example, the following diagram displays angles of 60- and 150-degrees.





It is often useful to think of angle as rotations, since we can then make sense of angles greater than 360°. We can also make sense of negative angles. On the blank circle to the right, locate points on the Ferris Wheel corresponding to angles of 135°, 720°, -90°, and -380°.

6) If we center our circle at the origin and construct it with a radius = 1 unit, we have the *unit circle*. We know that any point, (x, y), on the unit circle is 1 unit away from the origin.



From this circle, we can define a couple useful functions:

Given point P = (x, y) on the unit circle, we define:

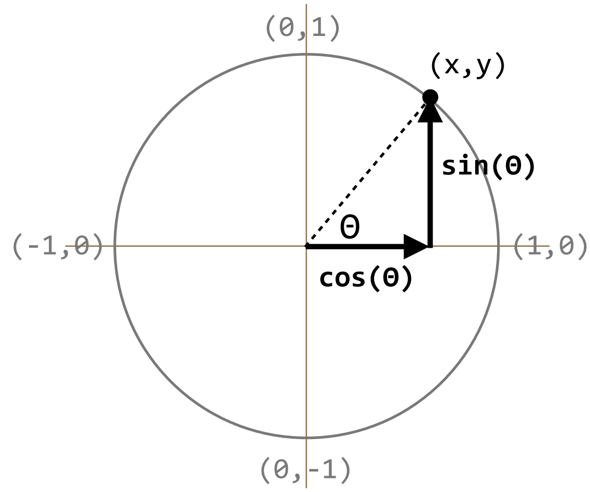
$$\cos(\theta) = x \quad \text{and} \quad \sin(\theta) = y$$

In other words, cos(Θ) represents the x-coordinate of a point on the unit circle specified by angle Θ and sin(Θ) represents the y-coordinate (or the height along the unit circle).

Use these definitions to evaluate the following:

sin(0) = _______ sin(90) = _______ sin(180) = _______ sin(270) = _______

cos(0) = _______ cos(90) = _______ cos(180) = _______ cos(270) = _______



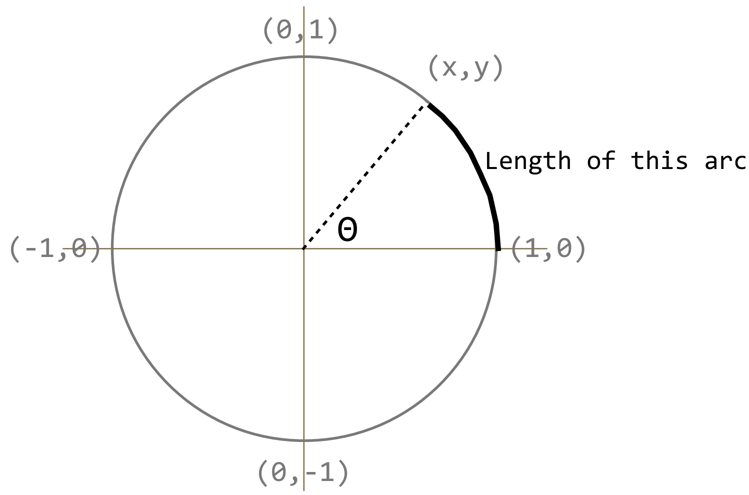
7) Use the above diagram and the table you just completed to sketch the graphs of the sine and cosine functions over the interval $0 \leq \Theta \leq 360$.

8) The point Q on the unit circle is specified by a 130° angle. Sketch the unit circle and locate this point. Then, use your calculator to find the coordinates of the point.

9) (Preview question) The point (0.90631, 0.42262) is located on the unit circle. Sketch a picture of this and use your calculator to find the measure of the angle specifying this point.

10) The Ferris Wheel has a radius of 312.5 feet. Find your height above ground as a function of the angle Θ measured from the x-axis. What is your height when the angle is 60°? What is your height at 240°?

11) So far, we've measured angles in degrees (rotations). It might be useful to have another way of measuring an angles that is based on the distance around a circle. An angle measured in *radians* is equal to the length of a corresponding arc of a unit circle.



Picture an arc around the entire circle. What would the length of this arc be? How many radians are equivalent to a 360° angle? How many radians are equivalent to a 90° angle?

12) If you can remember (or derive) the number of radians equivalent to 360°, you should be able to convert any angle from degrees-to-radians or radians-to-degrees. Convert the following angles:

Convert 3 degrees to radians:

Convert 3 radians to degrees:

Convert -280 degrees to radians:

13) We'll end this activity with a seemingly unrelated topic. As we'll see next time, we'll want to calculate the distance between any two points.

Plot the point (3, 4) on a graph and calculate the distance from this point to the origin.

Derive a general formula to calculate the distance from the point (x, y) to the origin.

Plot the points (3, 4) and (-2, 7). Calculate the distance between these two points.

Derive a general formula to calculate the distance between two points: (x_1, y_1) and (x_2, y_2).

14) Suppose we have point (x, y) on a unit circle. How far is this point from the origin? From what we derived above, what formula would we use to calculate this distance? Substitute sin(x) and cos(x) into this formula.