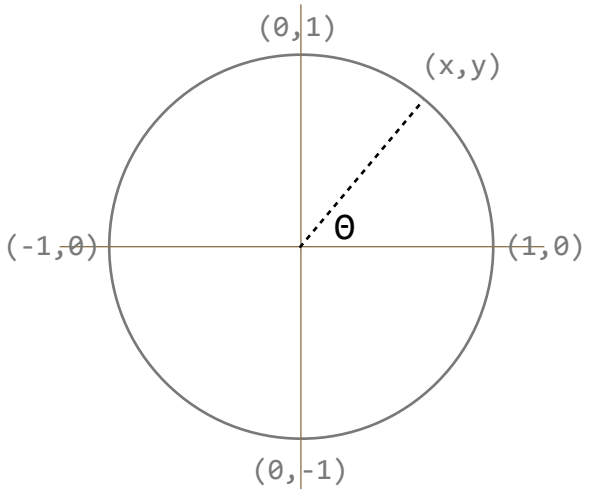
Activity #14: Trigonometric Functions (student learning outcomes listed in syllabus)

1) Let's begin by reviewing some trigonometric functions. Use the following unit circle to complete the table.



$$\sin(\theta) = \frac{y}{r} \qquad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{r}{y}$$

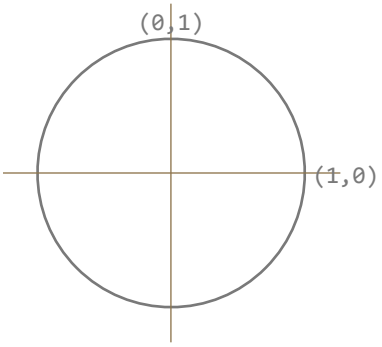
$$\cos(\theta) = \frac{x}{r} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{r}{x}$$

$$\tan(\theta) = \frac{y}{x} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{x}{y}$$

Degrees	Radians	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
0	0	0	1	0	--	1	--
30							
45							
60							
90							
120							
135							
150							
180	π						
210							
225							
240							
270							
300							
315							
330							
360	2π						

Suggestions:
A) Complete the table for 90, 180, and 270 degrees first.
B) Convert 45 degrees into radians. Use this information to convert 135, 225, and 315 degrees to radians
C) Convert 30 degrees into radians and then fill-in the radians column.

D) Let's evaluate the trigonometric functions at 45-degrees. To do this, locate the point (x, y) on the unit circle such that the point represents a 45-degree angle.



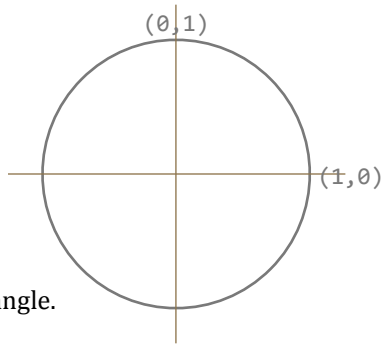
i) What is the distance from point (x, y) to the center of the circle?

ii) Draw a line from the origin to point (x, y). Write the equation of that line.

iii) Substitute the above equation into the formula used to find the distance between two points. Solve for y.

iv) If you don't like that, drop a vertical line down from your point (x, y) to the x-axis. What kind of triangle is this? How do you find the length of the sides on this triangle?

E) Let's evaluate the trigonometric functions at 60-degrees. To do this, locate the point (x, y) on the unit circle such that the point represents a 60-degree angle.



i) Draw a straight line from point (x, y) to the point (1, 0). What kind of triangle are you left with? What are the lengths of the sides of this triangle?

ii) Draw a vertical line from point (x, y) to the x-axis, dividing your triangle in half. You now have two congruent triangles. Find the lengths of the sides of each triangle. Use this information to complete the table.

F) Use what you just learned about a 60-degree angle to complete the table for the 30-degree angles.

2) In the last activity, we derived $\sin^2(\theta) + \cos^2(\theta) = 1$. Use this identity to derive two other useful trigonometric identities.

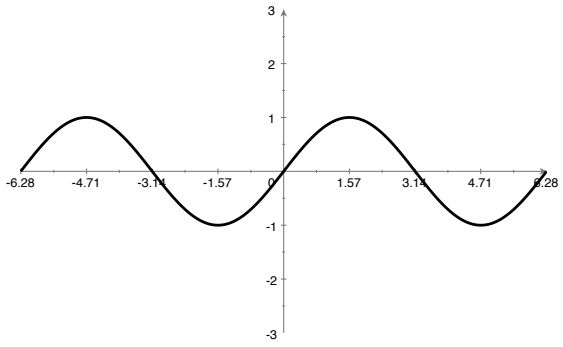
3) Sketch a graph of each trigonometric function over the interval [-2π, 2π]. Identify the domain and range of each function.

4) In the last activity, we modeled our height while riding a Ferris Wheel as a function of time. The graph of that function was simply a transformation of the sine function.

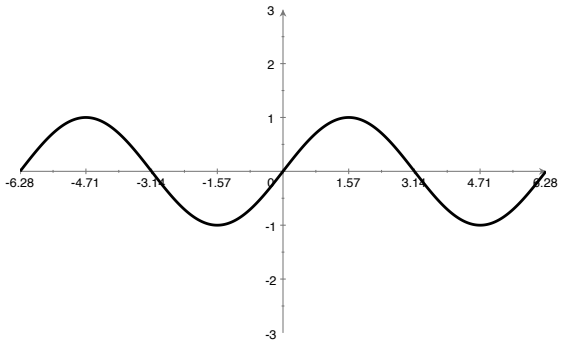
Transformations of the sine or cosine functions are called *sinusoidal functions*:

$$f(x) = A\sin B\left(x - h\right) + k \qquad f(x) = A\cos B\left(x - h\right) + k$$

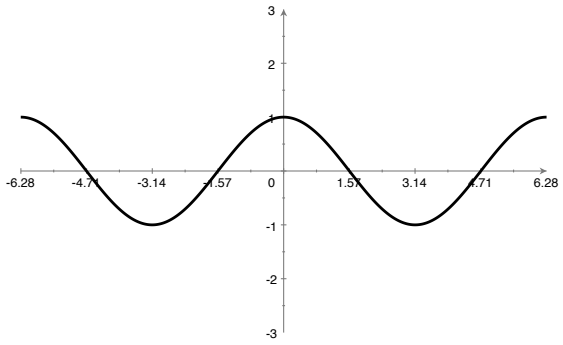
The sine and cosine functions are displayed below. Graph the given functions on the same axes and determine the effect *A* has on the graph of a sinusoidal function.



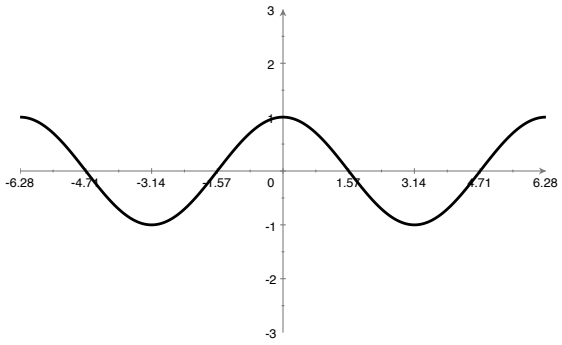
Graph: $2\sin(x)$



Graph: $-3\sin(x)$



Graph: $\frac{1}{2}\cos(x)$



Graph: $-\cos(x)$

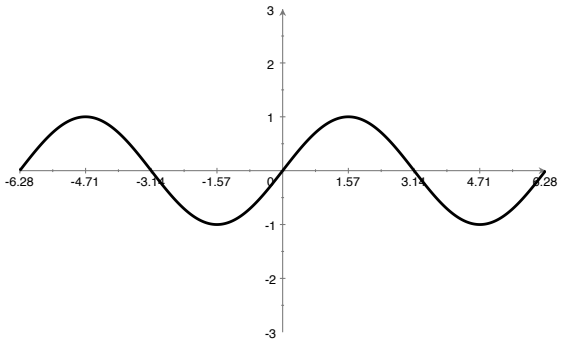
The **amplitude** of a sinusoidal function is equal to |A|. Identify how the following values of *A* transform a sinusoidal function:

- If A ≥ 1, _____

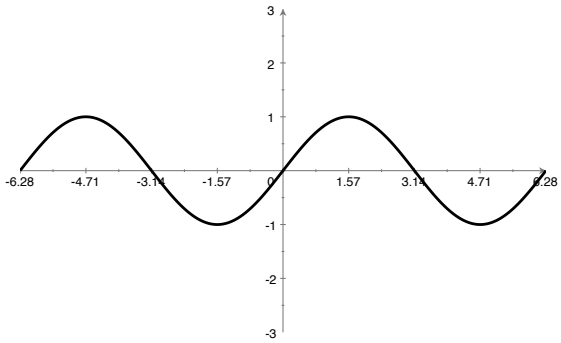
- If 0 ≤ A ≤ 1, _____

- If A < 0, _____

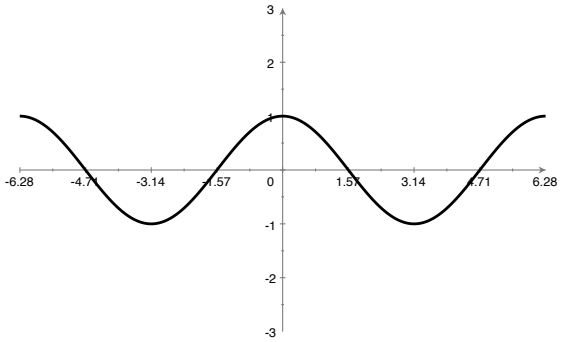
5) Graph the following functions and determine the effect of *k* on a sinusoidal function.



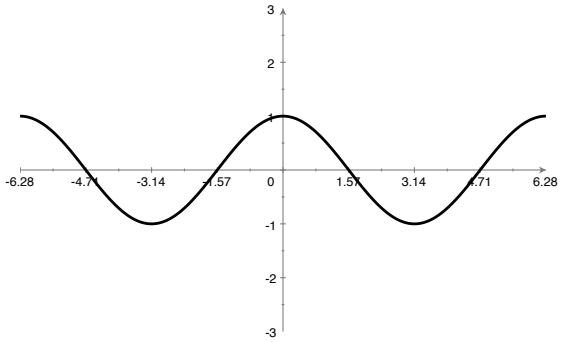
Graph: $\sin(x)+2$



Graph: $\sin(x)-1$



Graph: $\cos(x)+1$



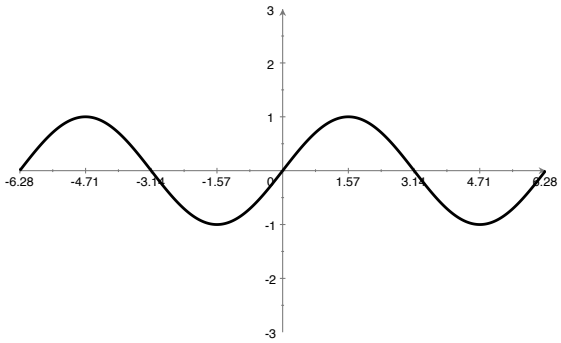
Graph: $-\cos(x)+1$

The **midline** of a sinusoidal function is the horizontal line y = k. Identify how the following values of *k* transform a sinusoidal function:

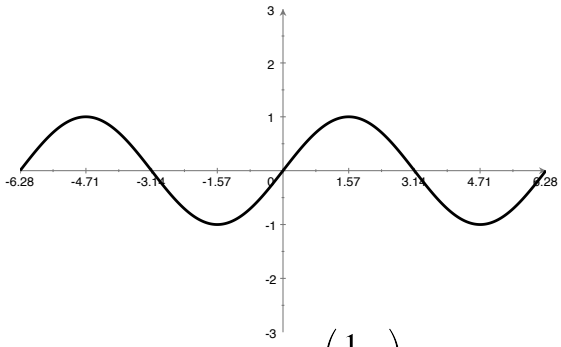
- If k > 0, _____

- If k < 0, _____

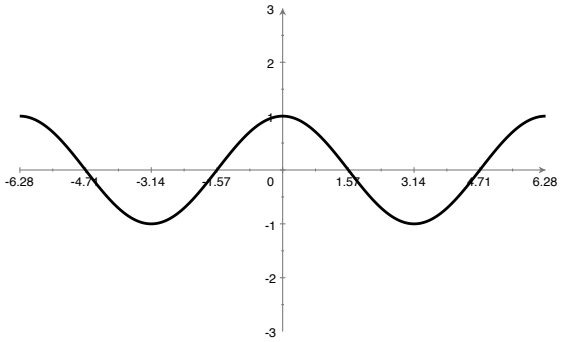
6) Graph the following functions and determine the effect of *B* on a sinusoidal function.



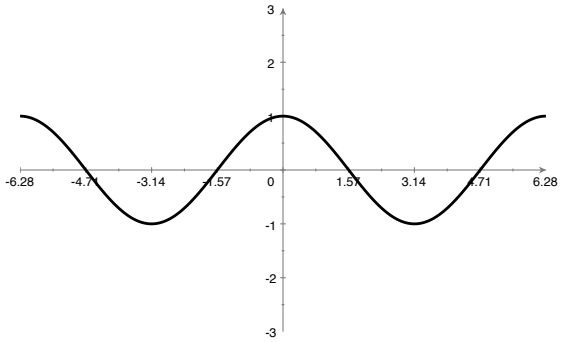
Graph: $\sin(2x)$



Graph: $\sin\left(\frac{1}{2}x\right)$



Graph: $\cos(4x)$

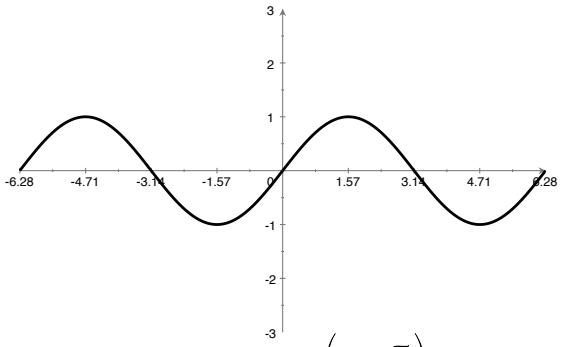


Graph: $\cos(-2x)$

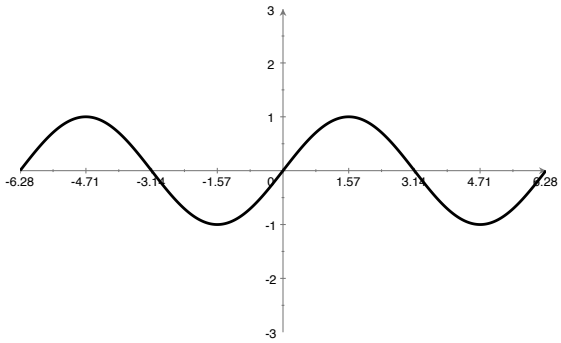
The **period** of a sinusoidal function is equal to $P = \frac{2\pi}{|B|}$. Identify how the following values of *B* transform a sinusoidal function:

- If B ≥ 1, _____
- If 0 < B < 1, _____
- If B < 0, _____

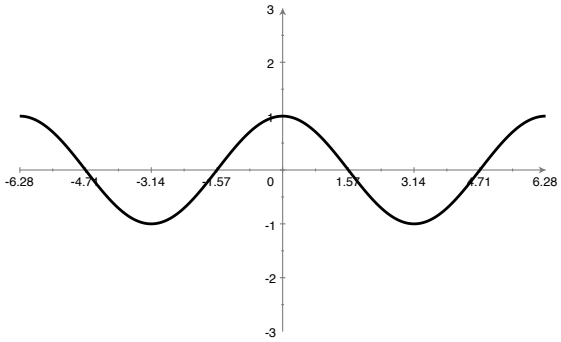
7) Graph the following functions and determine the effect of h on a sinusoidal function.



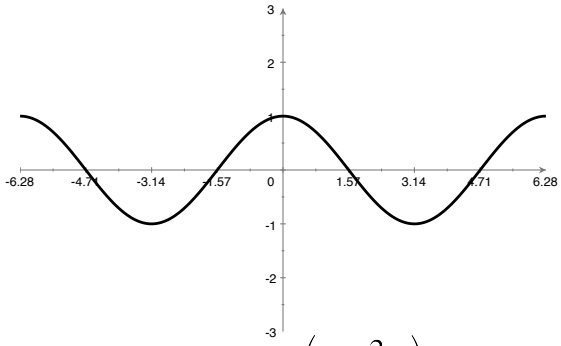
Graph: $\sin\left(x-\frac{\pi}{2}\right)$



Graph: $\sin(x+1)$



Graph: $\cos(x-\pi)$



Graph: $\cos\left(x+\frac{3\pi}{2}\right)$

- If h > 0, _____
- If h < 0, _____

8) Household electrical power in the US is provided in the form of alternating current (AC). Typically, the voltage cycles smoothly between +155.6 volts and –155.6 volts 60 times per second. Use a cosine function to model the alternating voltage.

A) First, write out the standard cosine function. Then calculate the amplitude this function should have. Modify A accordingly.

B) Next, determine the period this function should have and use that information to find B.

C) Write out the sinusoidal function and graph it on your calculator to check your answer.

9) A rabbit population in a national park rises and falls each year. It is at its minimum of 5000 rabbits in January. By July, as the weather warms up and foods grows more abundant, the population triples in size. By the following January, the population again falls to 5000 rabbits, completing the annual cycle. A trigonometric function models the size of the rabbit population as a function of time

A) First, write out the standard cosine function. Then calculate the amplitude this function should have. Modify A accordingly.

B) Next, determine the period this function should have and use that information to find B.

C) Determine if any other transformations (shifts or reflections) are needed. Write out the sinusoidal function and graph it on your calculator to check your answer.

D) Estimate when the rabbit population will equal 12,000.