

Topic #2: Linear Functions

Objectives

- Define linear function
 - Determine whether a dataset represents a linear function
 - Calculate the slope of a linear function
 - Find the formula for a linear function, given two points or a point and the slope, and write the formula in slope-intercept form
 - Interpret the slope and intercepts of a linear function
 - Determine the domain and range of a linear function and use this to create an appropriate graph
 - Solve systems of linear equations analytically and graphically
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1) ***When am I ever going to use this?*** I bet just about every student in a math class has asked that question at some point. Depending on your teacher, you may have gotten one of the following answers:*

You're going to use it... on the test, in another math class, in college

Maybe you'll become a... scientist, engineer, architect, or some other profession... and you'll need it

It's not important if you use it; we're building... critical thinking or problem solving... skills

When you get a job, you'll find you often need to do things you don't want to do

Today, we're going to use math to solve a real, important question:

_____ ?

2) Get into a group of 3 students. Together, you're going to work to answer this question. Whichever group gets the closest answer (without going over) will win some points.

Before we get started, get your group to agree on an estimated answer. Write that estimate here: _____

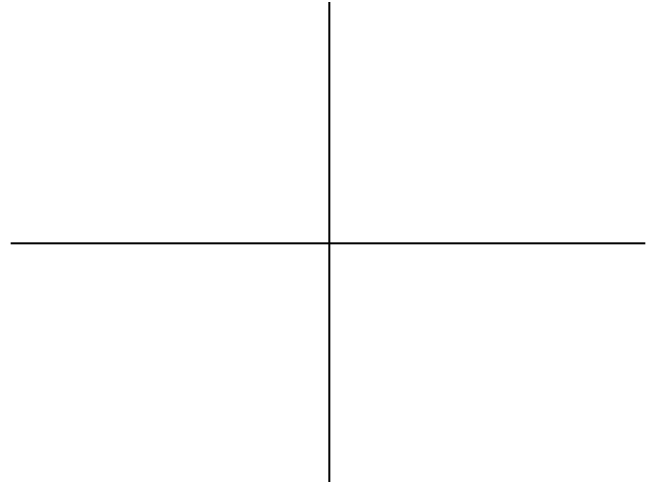
3) Is the height of a stack of cups a function of the number of cups in the stack? Explain your answer and identify:

Independent variable: _____ Dependent variable: _____

Domain: _____ Range: _____

4) Create a table to display the height of a stack of 1, 2, 3, 4, 100, and c cups. Use this table to create a graph.

Cups	Height (cm)	Increase in height
1		
2		
3		
4		
100		(N/A)
c		(N/A)

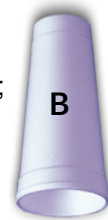


5) Suppose we stacked different types of cups. How many cups of type A would we need to stack to reach 185 cm? How many cups of type B would we need? Draw graphs representing each scenario.

The entire cup is 10cm tall;
the lip is 4cm tall.



The entire cup is 18cm tall;
the lip is 1cm tall.



6) Suppose we started stacking Type A cups on the left side of a table and Type B cups on the right side. How many cups would it take for the Type A stack to rise above the Type B stack?

7) I have a great idea for an iPhone app. It's so great that I can't even tell you what it is. What I can tell you is that once I actually develop the app and put it on the App Store, I'm certain to become a millionaire.

- To get an app in the App Store, I need to:
- Pay \$99 to join the iOS Developer Program for a year
 - Develop the app and upload it to the App Store
 - Choose the price I want to sell the app for (from 99¢ to \$999.99)

For app I sell, Apple gets 30% of the revenue and I get to keep 70%.

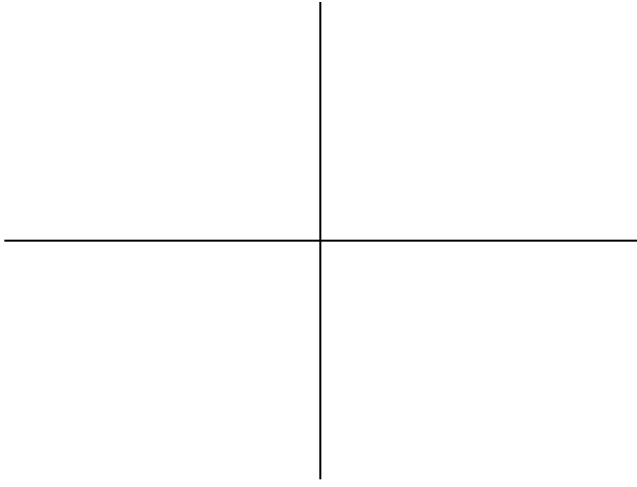
If it costs me \$200 to develop the app, is the profit I receive from my app a function of the number of copies sold?

Independent variable = _____ Dependent variable = _____

Reasonable domain = _____ Reasonable range = _____

8) Fill-in the following table to show my profits if I sell 0, 1, 2, 3, x, 1000, and 100,000 copies of my app. Use the table to create a graph.

Apps sold	Profits	Change in profit
0		
1		
2		
3		
x		(N/A)
1000		(N/A)
100,000		(N/A)



This is an example of a **linear function**.

We can write linear functions in slope-intercept form as: $f(x) = mx + b$, where m = slope and b = y-intercept

What does it mean for a function to be linear? _____

What does the slope of a function represent? _____

How do we calculate the slope of a linear function? _____

What does the y-intercept of a linear function represent? _____

How do we calculate the y-intercept of a linear function? _____

9) How much profit would I make if I sold 15,000 copies of my app at \$1 apiece?

10) How many copies would I need to sell at \$1 apiece to make \$25,000 in profit?

11) We're going to construct linear functions to model simple scenarios. In this first example, we'll be given the slope and y-intercept.

Immediately following a tanker spill, a scientist measures 96 tons of contaminant in a lake. The scientist estimates that 7 tons of the contaminant will dissipate every hour.

- a) Will a linear function accurately model the amount of contaminant as a function of time?
- b) Write out the formula for this linear function in slope-intercept form.
- c) How long will it take before the lake is free from contaminant? Your answer to this question is the x-intercept of this function. What does an x-intercept represent?
- d) Sketch a graph of this function.
- e) How much contaminant will remain in the lake after 5 hours? 15 hours?
- f) What is the domain of our function? What is the range?
- g) When will the lake have 47 tons of contaminant?

12) This time, we'll find a linear function given two points. Here's the scenario:

According to the MPAA, the average price of a movie ticket in 1964 was \$0.93. The average in 2014 was \$8.15.*
<http://www.boxofficemojo.com/about/adjuster.htm>

a) Identify the independent and dependent variables and write out the two given points as ordered pairs: (x, y).

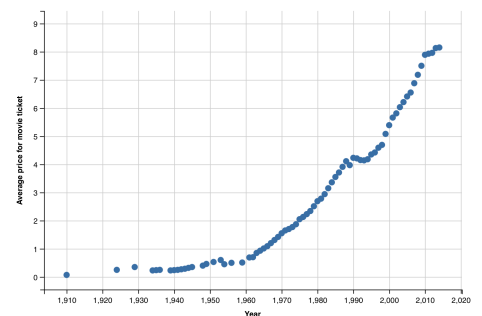
b) Calculate the slope of the line that would connect those two points.

c) Write the formula you used to calculate the slope. Manipulate it to derive the point-slope form of a line.

d) Now substitute one of the given points into the point-slope form to find the formula for this linear function. Sketch a graph of this linear function.

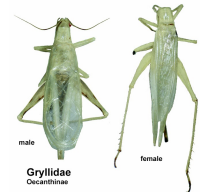
e) If this linear model holds in the future, in what year will movie tickets cost \$10, on average? Solve this graphically and analytically.

f) According to your linear model, what was the average price of a ticket in 1924? According to the MPAA, the actual average price was \$0.25. Evaluate how well the linear function models this scenario.



13) We can also find linear functions quickly using the point-slope form if we're given the slope and a single point.

Most crickets chirp at a rate that increases in warmer temperatures. For *Oecanthus fultoni* (the snowy tree cricket), the relationship between temperature and chirp rate is so reliable that this type of cricket is called the thermometer cricket.



When the temperature is 80°F , the crickets chirp 60 times per minute. For every 12° increase in temperature, the chirp rate increases by 3 chirps per minute

a) Write out the point and calculate the slope that we are given in this scenario. What does the slope represent?

b) Sketch a graph of this function and find the formula. What are the domain and range of this function?

c) According to our linear model, what chirp rate should we expect if it is 32° -degrees?

d) Suppose we hear a chirp rate of 52 chirps per minute. According to our model, what is the temperature?

14) Suppose I need to rent a moving truck. U-Haul charges \$29.95 plus 59¢ a mile, while Budget charges \$24.95 plus 99¢ per mile. Which company should I choose?

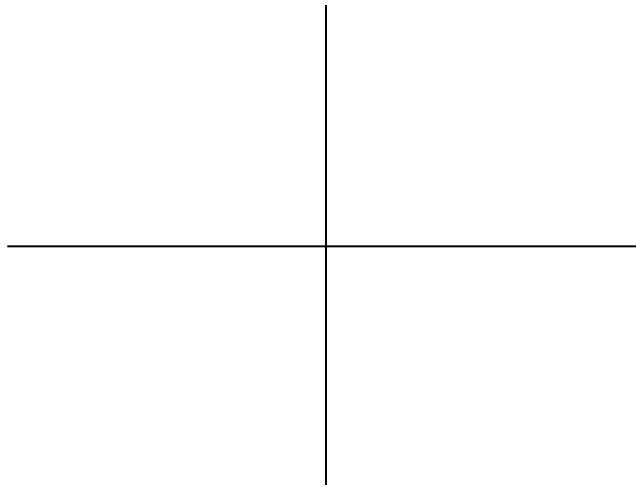
15) Sketch and label graphs of the following linear functions:

$$f(x) = 2x + 4$$

$$g(x) = 2x - 4$$

$$h(x) = -2x + 4$$

$$i(x) = 0x + 4$$



16) While Clark Kent attends a Pulitzer Prize ceremony for Lois Lane, Lex Luthor steals an experimental aircraft from NASA headquarters. Knowing Superman would quickly find out, Lex flies towards his kryptonite-defended base at a speed of mach 6.7 (5,100 mph).

2 hours later (after the ceremony ends), Superman arrives at NASA headquarters and learns about the missing aircraft. He quickly flies after Lex at a speed of 6,600 mph.

If Lex Luthor's secret base is 50,000 miles away from NASA headquarters, will Superman catch him in time?

a) Find the formula to model Lex Luthor's distance from NASA headquarters as a function of time.

b) How far away is Lex when Superman begins his pursuit?

c) How long will it take for Lex to reach his kryptonite-defended base?

d) Find the formula to model Superman's distance from NASA headquarters as a function of time?

e) Will Superman catch Lex before he reaches his base? Solve graphically and analytically?

17) You need to choose a smartphone.

Verizon charges \$40 for the phone, \$50 for 1GB per month, and \$12 for every 1GB you go over that limit.
AT&T charges \$30 for the phone, \$40 for 1GB per month, and \$15 for every 1GB you go over that limit.
Sprint charges \$35 for the phone and a flat rate of \$80 per month for unlimited data.

Construct a linear function for each company that models a customer's bill as a function of how many GBs of data they use. Which company has the cheapest plan?