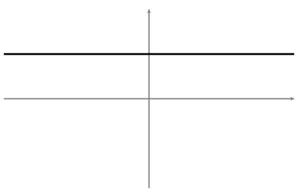
Topic #5: Transformations and Piecewise Functions

Objectives

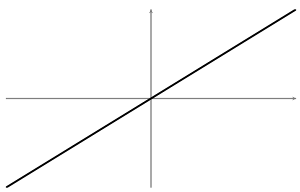
- Translate piecewise functions from formulas-to-graphs and graphs-to-formulas
- Identify the domain and range of piecewise functions
- Generate a piecewise function to model a situation
- Graph the effect of a transformation (shift, compression/stretch, reflection, absolute value) on an elementary function
- Given a graph, write out the formula using elementary functions and transformations
- Use transformations to model a scenario using elementary functions

In activity #1, we learned the concept of a function and how they could be used to model data.
In activities 2-3, we modeled situations using linear functions.
In this activity, we'll learn general methods for combining and transforming function to create mathematical models.

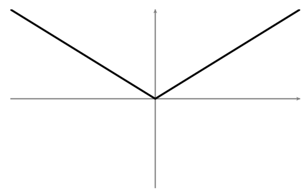
Let's begin by taking a look at the elementary functions we will review in this class:



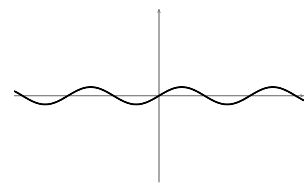
Constant: $f(x) = c$
Domain: $(-\infty,\infty)$
Range: $[c]$



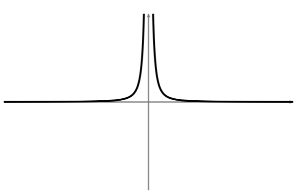
Identity: $f(x) = x$
Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



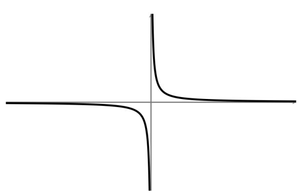
Absolute value: $f(x) = |x|$
Domain: $(-\infty,\infty)$
Range: $[0,\infty)$



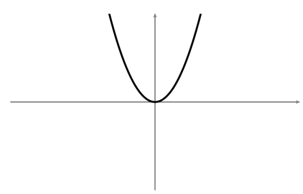
Sine: $f(x) = \sin(x)$
Domain: $(-\infty,\infty)$
Range: $[-1,1]$



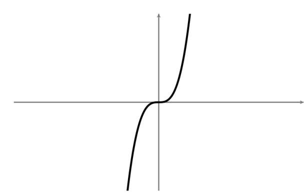
Reciprocal squared: $f(x) = \dfrac{1}{x^2}$
Domain: $(-\infty,0)\cup(0,\infty)$
Range: $(0,\infty)$



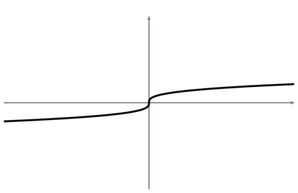
Reciprocal: $f(x) = \dfrac{1}{x}$
Domain: $(-\infty,0)\cup(0,\infty)$
Range: $(0,\infty)$



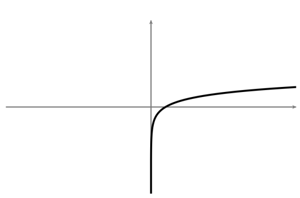
Quadratic: $f(x) = x^2$
Domain: $(-\infty,\infty)$
Range: $[0,\infty)$



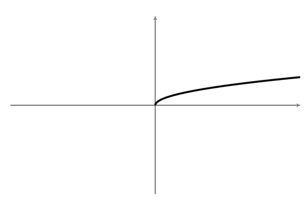
Cubic: $f(x) = x^3$
Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



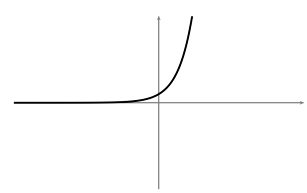
Cube root: $f(x) = \sqrt[3]{x} = x^{1/3}$
Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



Logarithmic: $f(x) = \log(x)$
Domain: $(0,\infty)$
Range: $(-\infty,\infty)$



Square root: $f(x) = \sqrt{x} = x^{1/2}$
Domain: $[0,\infty)$
Range: $[0,\infty)$



Exponential: $f(x) = 10^x$
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$

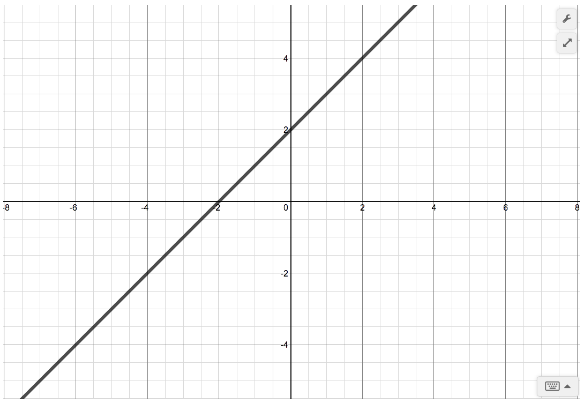
Now, when we examine a scatterplot, we can pay attention to the shape of the data. We can then combine, shift, stretch, and reflect these elementary functions to come up with our mathematical model.

Let's get started by examining the identity function (the second one displayed above) and how it relates to the linear functions we've already studied.

1) We're going to play a little game. Keep in mind it's a *math game*, so don't get your hopes up too high.

Using desmos.com, I'll show you the graph of a function. Your task will then be to graph a series of transformations to that function.

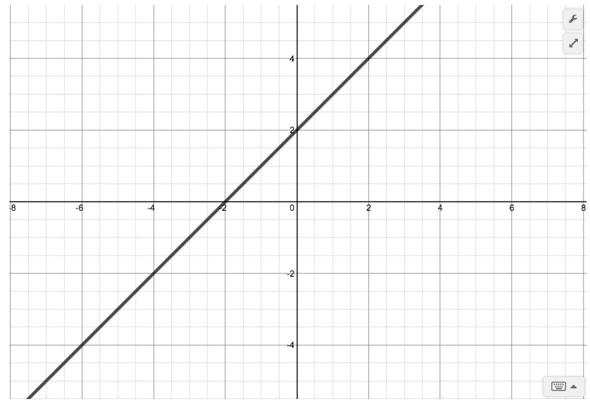
We'll start with a linear function: f(x) = mx + b. On the same set of axes, sketch the following:



$a:\ -f(x)$

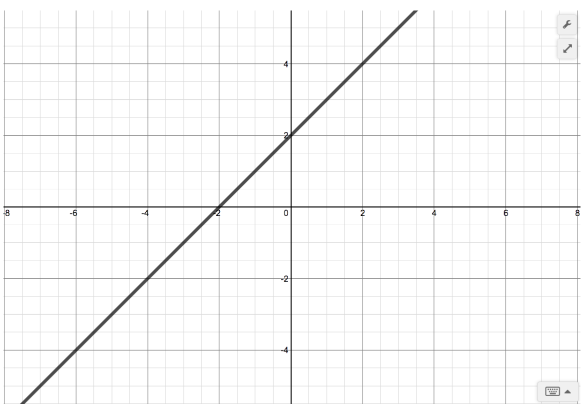
$b:\ 3f(x)$

$c:\ \dfrac{1}{2}f(x)$



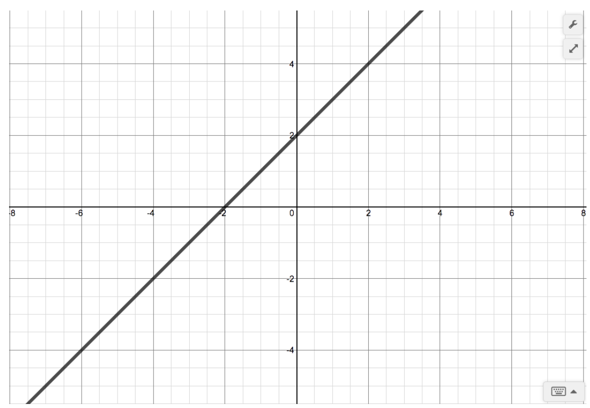
$d: f(x)+2$

$e: f(x)-4$



$f: f(x+2)$

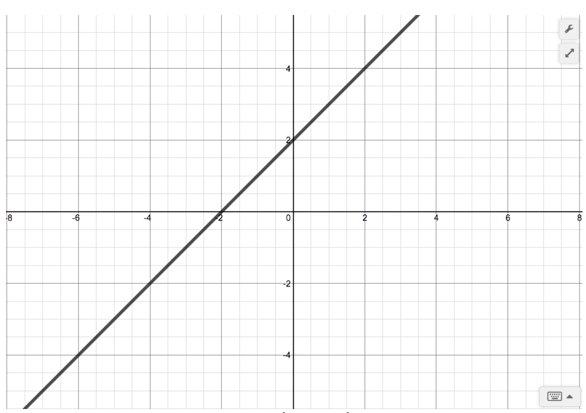
$g: f(x-4)$



$h: f(-x)$

$i: f(3x)$

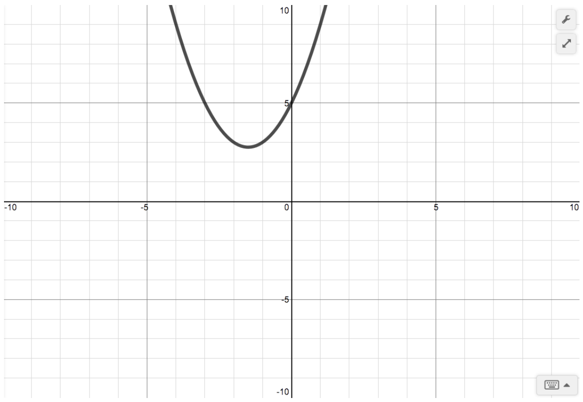
$j: f\left(\dfrac{1}{2}x\right)$

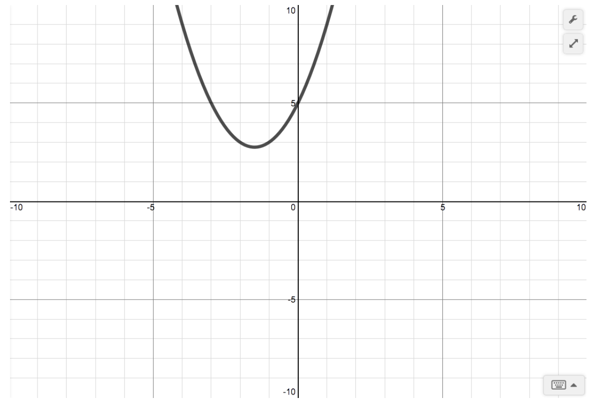


$k:\ |f(x)|$

$l: f(|x|)$

2) Now let's try a quadratic





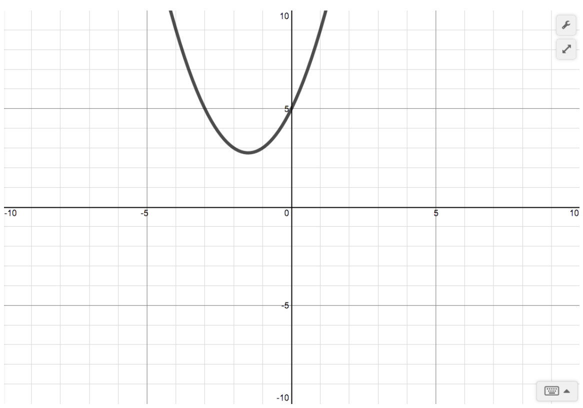
$a:\ -f(x)$

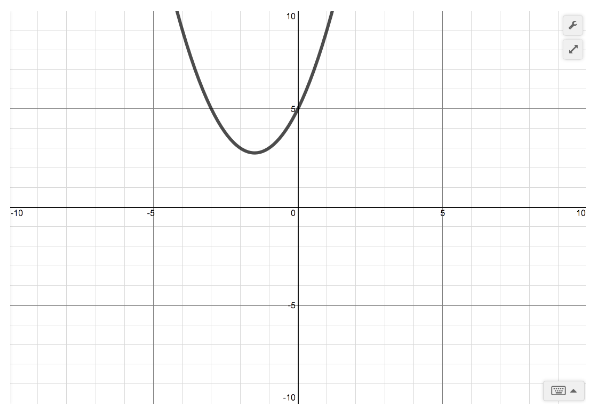
$b:\ 3f(x)$

$c:\ \frac{1}{2}f(x)$

$d: f(x)+2$

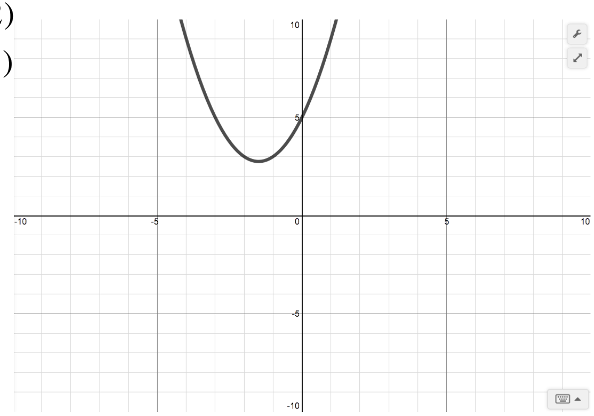
$e: f(x)-4$





$f: f(x+2)$

$g: f(x-4)$



$h: f(-x)$

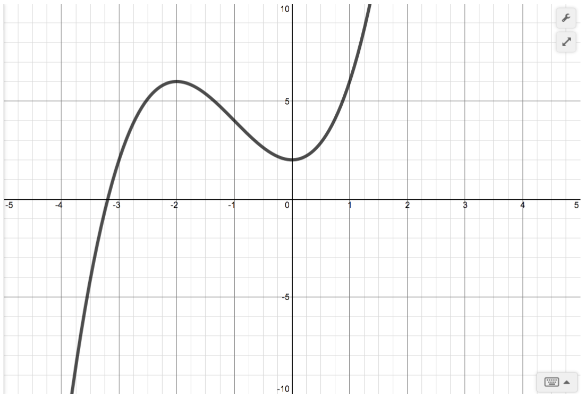
$i: f(3x)$

$j: f\left(\frac{1}{2}x\right)$

$k:\ |f(x)|$

$l: f(|x|)$

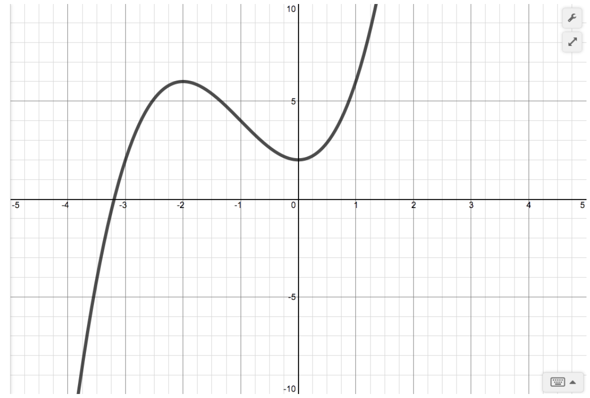
3) A cubic function…



$a: \ -f(x)$

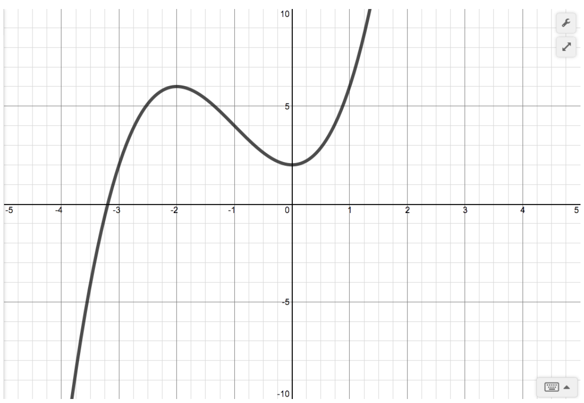
$b: \ 3f(x)$

$c: \ \frac{1}{2}f(x)$



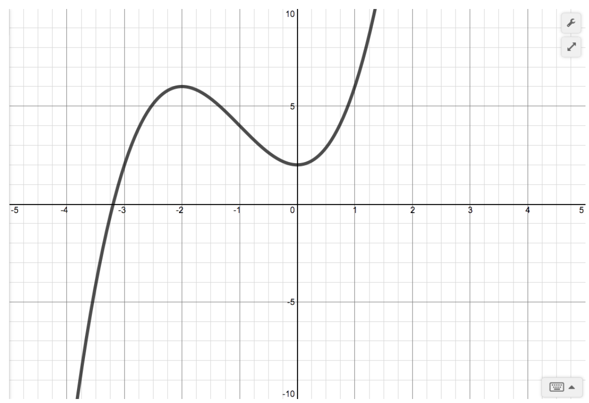
$d: f(x)+2$

$e: f(x)-4$



$f: f(x+2)$

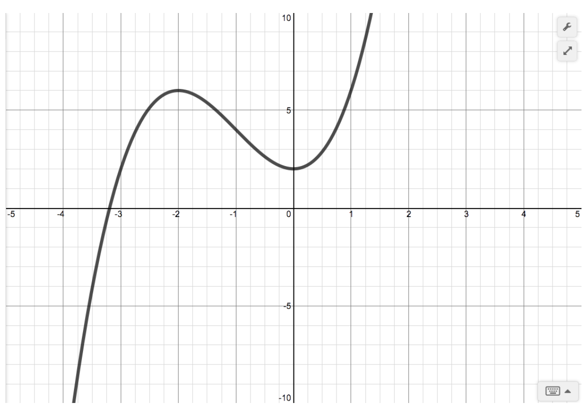
$g: f(x-4)$



$h: f(-x)$

$i: f(3x)$

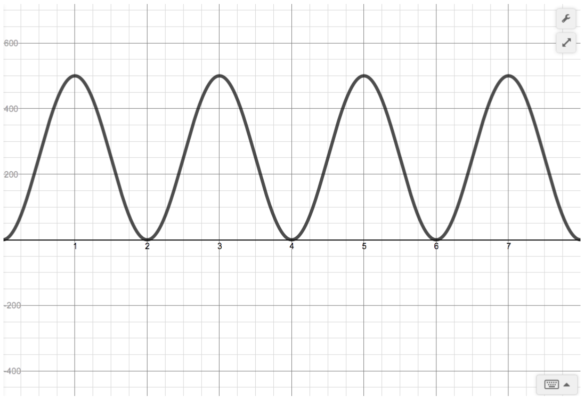
$j: f\left(\frac{1}{2}x\right)$



$k: \ |f(x)|$

$l: f(|x|)$

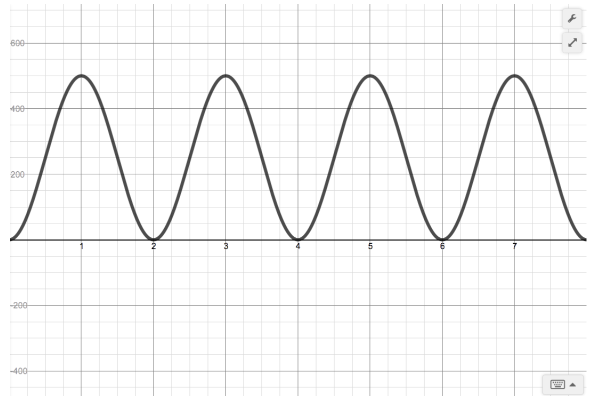
4) Finally, a periodic function…



$a:\ -f(x)$

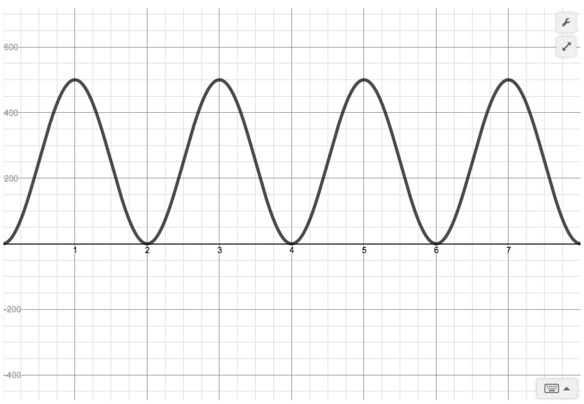
$b:\ 3f(x)$

$c:\ \frac{1}{2}f(x)$



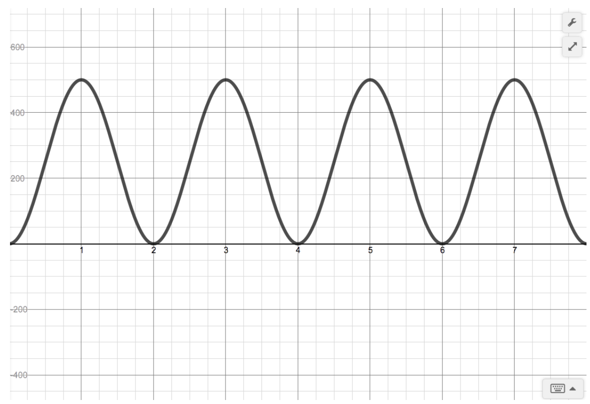
$d: f(x)+200$

$e: f(x)-400$



$f: f(x+2)$

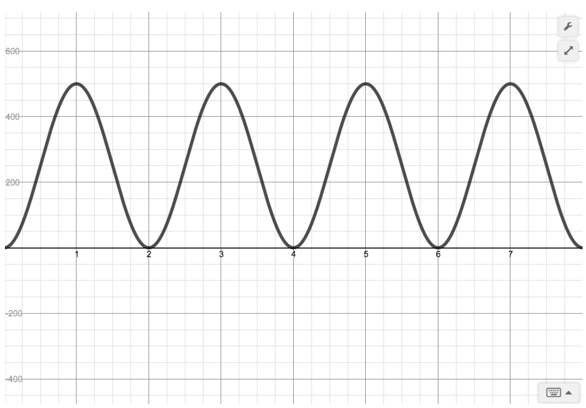
$g: f(x-4)$



$h: f(-x)$

$i: f(3x)$

$j: f\left(\frac{1}{2}x\right)$

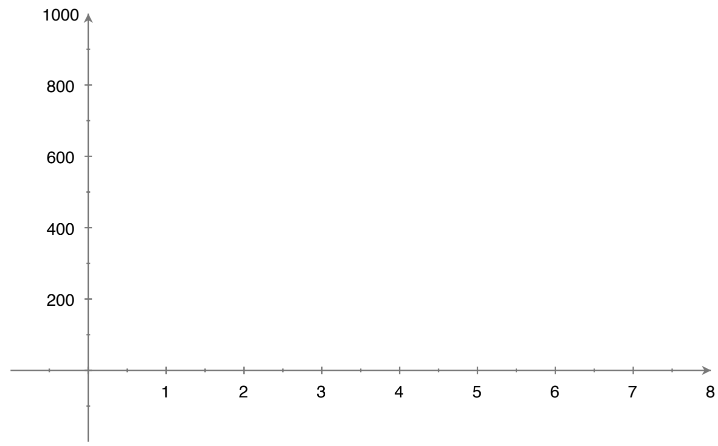


$k:\ |f(x)|$

$l: f(|x|)$

5) When riding a Ferris Wheel, your height is a function of time. You start at a height of zero and reach a maximum of 500 feet. The Ferris Wheel completes one whole rotation every 2 minutes and the ride lasts 8 minutes.

Sketch a graph of your height as a function of time.



If f(x) represents your height on the Ferris Wheel at any given time, match the following functions to their explanations. If it helps, complete the table.

A Ferris Wheel that runs twice as fast

A Ferris Wheel that is only half as tall

A Ferris Wheel that is twice as tall

A Ferris Wheel that runs only half as fast

$$2f(x)$$

$$f(2x)$$

$$\frac{1}{2}f(x)$$

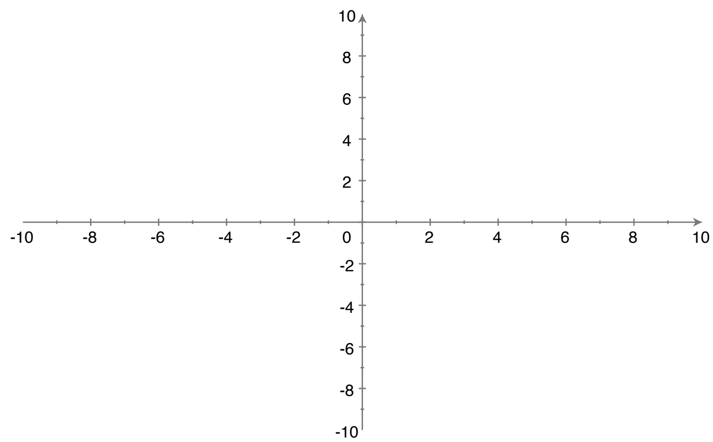
$$f\left(\frac{1}{2}x\right)$$

x	0	0.5	1	1.5	2	3	4
f(x)	0	250	500	250	0	500	0
2f(x)	______	______	______	______	______	______	______
f(2x)	______	______	______	______	______	______	______
(1/2)f(x)	______	______	______	______	______	______	______
f(x/2)	______	______	______	______	______	______	______

6) Another way to create a new function to model data is by combining functions. A piecewise function is a collection of functions that apply to certain intervals of the domain.

Let's look at an example of a piecewise function. Graph f(x) on the axes below.

$$f(x) = \begin{cases} 3x+1 & ,x \le 0 \\ 1 & ,0 < x < 4 \\ (x-3)^2 & ,x \ge 4 \end{cases}$$



To graph this function on your calculator, you need to enter the following: $Y_1 = (3x+1)*(x \le 0)$

$Y_2 = (1)*(x > 0)(x < 4)$

Note: The inequality symbols are listed under TEST (above MATH) $Y_3 = ((x-3)^2)*(x \ge 4)$

7) In a triathlon, an athlete's distance is a function of time. Suppose a triathlon consists of the following:

You swim for 2.4 miles at 2 mph
You ride a bike for at 20 mph for 112 miles
You run a 26.2 mile marathon at a speed of 9 mph

Your goal is to graph the distance of the triathlete as a function of time and write out the piecewise function.