Activity #7: Exponential Functions (student learning outcomes listed in syllabus)

1) Recall that in the last activity, we wasted our three wishes on money machines (and an adapter). Suppose Al, the next person who finds the magic lamp wishes for something important. Suppose Al wishes for his own private island.

Instantly, Al gets whisked away to his tropical paradise only to find that it is inhabited by snakes and rabbits. As Al walks around his island, he happens to see a snake attack and kill a rabbit. Saddened, Al uses his second wish to wish that rabbits can never get sick or die on his island. He uses his third wish to banish all snakes from his island forever.

Having used all 3 wishes, Al decides to explore his island. Walking around the entire island, Al counts 100 rabbits.

One month later, when he was bored enough to start counting rabbits again, Al counts 110 rabbits*. With nothing better to do, Al decides to create a mathematical model of the population growth of rabbits on his island.

Let's first assume that a linear function models the number of rabbits each month. Complete the table and sketch a graph.

Month	Number of rabbits	Change from previous month
0	_____	
1	_____	_____
2	_____	_____
10	_____	_____
60	_____	_____
61	_____	_____
M	_____	_____

2) Explain why this is not a realistic model for the population growth of rabbits. How could you improve the model?

3) Instead of assuming the number of rabbits increases by a constant rate (100) each month, let's assume the rabbits increase by a constant percentage each month. Complete the table and sketch a graph.

Month	Number of rabbits	Change from previous month
0	_____	
1	_____	_____
2	_____	_____
10	_____	_____
60	_____	_____
61	_____	_____
M	_____	_____

* The gestation period of a rabbit is only 30 days, so they would be increasing in number every month.
Source: http://en.wikipedia.org/wiki/Rabbit

4) Write out the formula you found to estimate the number of rabbits at month *m*. Identify the domain and range of this function.

5) Lady Gaga is currently (as of November 19, 2012) the most popular Twitter user. In October of 2011, Forbes magazine noted that Lady Gaga had 15 million followers on Twitter and that the number of followers was increasing by 6% each month.

Complete the table showing the number of Twitter followers Forbes magazine predicted for Lady Gaga each month. Write out a formula to determine the number of her followers as a function of the month.

Source: http://www.forbes.com/sites/meghancasserly/2011/10/28/lady-gagas-newest-record-15-million-twitter-followers/

Month	Followers	Change from previous month
0	15 million	
1	_____	_____
2	_____	_____
12	_____	_____
24	_____	_____
M	_____	_____

6) May 18, 2012 marked the first day the public could buy stock in Facebook. At the beginning of that day, each share of Facebook stock was worth $38. Six months later Facebook stock was worth $22.90 per share. This represents a decline of $15.10 per share, or 39.7%, over 6 months.

On average, by what percent did Facebook stock decline each month? Develop a formula modeling Facebook stock as a function of time (months since May 2012).

An **exponential function** is defined by $f(t) = a(1+r)^t$ where a > 0

7) Explain what the parameters *a, r,* and *t* represent in an exponential function. Why must a > 0?

8) Sketch graphs of the following exponential functions. What impact do the parameters *a* and *r* have on the graph?

$f(t)=3(1+.1)^t$ $g(t)=5(1+.1)^t$ $h(t)=3(1+.3)^t$ $i(t)=3(1-.1)^t$

9) Before we continue, I want to make sure you can handle percentages without any problems.

a) What is 12% of 475?

b) If we start with $1000 in 2010 and end with $3000 in 2012, what is the total percentage change? What is the annual percentage change?

c) If we start with $1000 in 2010 and end with $380 in 2012, what is the total percentage change? What is the annual percentage change?

d) If we multiply a number by 1.15, it increases by _____%.

e) If we multiply a number by 0.72, it increases by _____ %.

10) In 1998, the population of the U.S. was about 268.2 million with an annual growth rate of 0.7%. At the same time, the population of Mexico was 100.1 million with an annual growth rate of 2.2%.

Write out exponential models for the population growth of each country. Then, assuming these growth rates do not change, use graphical methods to estimate when the population of Mexico would equal the population of the U.S.

11) After graduating from SAU, you're given a job with a starting salary of $40,000. If you're promised 3% raises annually, when will you make more than $100,000 per year?

12) In question #5, we wrote out the exponential function that models the number of Lady Gaga followers on Twitter over time. If the U.S. has 268.2 million people with an annual growth rate of 0.7%, when will Lady Gaga followers outnumber people in the U.S.?

13) In 1990, 5.01 billion metric tons of carbon dioxide were emitted into the atmosphere in the U.S. In 2002, 5.80 billion metric tons were emitted. Write an exponential function to model the amount of carbon dioxide emitted as a function of the number of years since 1990. Use this model to estimate the amount emitted in 2010 and 1980.

14) A cup of coffee contains 100 mg of caffeine. Each hour, your body metabolizes 16% of the caffeine. Suppose you drink two cups of coffee at 8:00 am. How much caffeine remains in your body at 9:00 am, noon, and 8:00 pm? When will you have 50 mg of caffeine in your body? When will you have 25 mg? When will all the caffeine be eliminated from your system? Graph this function.

15) Cobalt-60 has a half-life of 5.27 years. The radioactive decay of Cobalt-60 is modeled by the function $C(t) = C_0e^{kt}$,

where: C_0 represents the initial amount of Cobalt-60
t represents time
k is a constant of decay (a negative number)
e is the natural base

Suppose a cobalt bomb is set-off in the desert, making the area uninhabitable. If the residual Cobalt levels are 50 times higher than safe levels, how long will it take before people can live in that area?

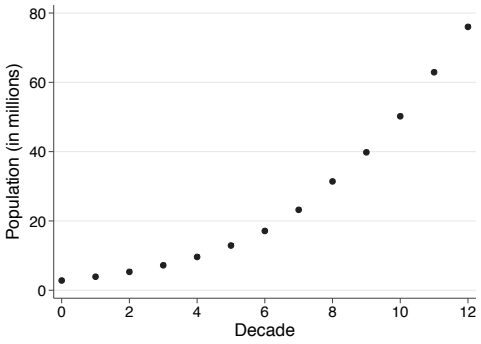
Hint: Use the half-life information to find k. Then, solve the rest of the problem.

Source: http://en.wikipedia.org/wiki/Cobalt_bomb

16) In activity 3, we introduced the concept of linear regression for finding the best linear function to fit a set of data. We'll now learn how to fit exponential functions to data that suggest an exponential relationship.

Let's start with a simple example. Let's see if we can model the following U.S. population growth.

Decade	Year	Population
0	1780	2.8
1	1790	3.9
2	1800	5.3
3	1810	7.2
4	1820	9.6
5	1830	12.9
6	1840	17.1
7	1850	23.2
8	1860	31.4
9	1870	39.8
10	1880	50.2
11	1890	62.9
12	1900	76.0



Let's enter our data into our calculators.

```
Enter the STAT menu
We want to enter data, so select EDIT...
Enter the data into the lists L1 and L2, pressing ENTER after each entry
QUIT to leave the data entry screen
```

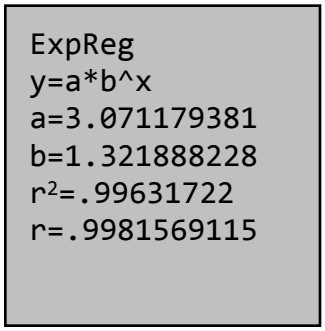
Before we calculate the best-fitting line, we can display a scatterplot.

```
Press [2ND] [Y=] to access the STAT PLOT editor
Press [ENTER] to edit Plot1
Press [ENTER] to turn ON Plot1
Scroll down and highlight the scatterplot graph type (first option in the first row)
Press [ENTER] to select the scatterplot
Make sure XList is set to L1 and Ylist is set to L2.
Press [ZOOM][9] to perform a ZoomStat and display your scatterplot
```

To find the line of best fit, you'll need to...

Press STAT to enter the statistics menu
Move right to highlight the CALC menu
Select EXPREG to calculate a linear regression
(by default, this command treats L1 as the independent variable and L2 as the dependent variable)
Press ENTER twice to calculate the least-squares regression line

You should see this screen...



17) Write out the formula for this exponential function. Interpret the numbers in the formula and graph it on top of your scatterplot. What does the r^2 value represent? Does this function fit our data well? What makes this exponential function the "best" exponential function for this data?

18) The following tables display the number of violent crimes per 100,000 people in the U.S. from 1990-2011.

Year	Year*	Violent Crimes
1990	0	731.8
1991	1	758.1
1992	2	757.5
1993	3	746.8
1994	4	713.6
1995	5	684.5
1996	6	636.6
1997	7	611.0
1998	8	566.4
1999	9	523.0
2000	10	506.5

Year	Year*	Violent Crimes
2001	11	504.5
2002	12	494.4
2003	13	475.8
2004	14	463.2
2005	15	469.0
2006	16	473.6
2007	17	466.9
2008	18	457.5
2009	19	431.9
2010	20	404.5
2011	21	386.3

Source: http://www.disastercenter.com/crime/uscrime.htm

Enter this data into your calculator and do the following:

a) Create a scatterplot and sketch it below. Do you think a linear or exponential function will better fit this data?
b) Find the best fitting linear function and exponential function. Write their formulas below and record the r^2 values.
c) Graph each function on top of your scatterplot.
d) Use both models to predict the number of violent crimes (per 100,000 people) in 2012.
e) With both models, predict when the number of violent crimes will reach 300 per 100,000 people.