Activity #9: Polynomial Functions

(student learning outcomes listed in syllabus)

1) A Roth IRA (individual retirement account) is a personal savings plan that allows individuals to contribute up to $5000 per year. Since taxes have already been paid on the contributions, an individual's money grows tax-free. Suppose you put $5000 into a Roth IRA and invest that money in an S&P 500 index fund. If that money grows 7% each year (the average return from 1950-2009), how much money would you have in 5 years? How much would you have in 40 years?

Source: http://www.simplestockinvesting.com/SP500-historical-real-total-returns.htm

2) This time, suppose you put $5000 into this Roth IRA each year. If your money grows 7% each year and you contribute $5000 each year, how much money will you have in 5 years? How much would you have in 40 years?

Let's get started by creating a table.

Year	Amount of money in Roth IRA
t=0*	
t=1	
t=2	
t=3	
t=4	
t=5	

3) What annual interest rate must your money earn in order to have $40,000 in 5 years? Go ahead and guess.

4) Since we don't know the interest rate, let's assign it the variable *x*. The following table shows how much money we would have each year if we earned 100(x-1)% interest annually.

Year	Amount of money in Roth IRA
t=0*	5000
t=1	$5000x + 5000$
t=2	$5000x^2 + 5000x + 5000$
t=3	$5000x^3 + 5000x^2 + 5000x + 5000$
t=4	$5000x^4 + 5000x^3 + 5000x^2 + 5000x + 5000$
t=5	$5000x^5 + 5000x^4 + 5000x^3 + 5000x^2 + 5000x + 5000$

Graph the function modeling the amount of money after 5 years. Describe the graph (is it increasing/decreasing, concave up/down?). Estimate the interest rate needed to earn $40,000.

A polynomial function of degree *n* has the form: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$.

5) An oil pipeline bursts in the Gulf of Mexico, causing an oil slick in a roughly circular shape. The slick is currently 24 miles in radius, but that radius is increasing by 8 miles each week.

Write a formula expressing the radius, *r*, of the spill as a function of the number of weeks, *w*, that have passed.

r(w) =

Write a formula expressing the area of a circle (A) as a function of the radius (r).

A(r) =

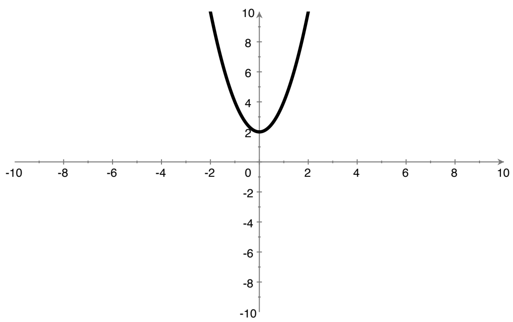
Compose these functions to express the area of the spill as a function of the number of weeks that have passed.

A(r(w)) = A(w) =

Estimate the number of weeks before the spill has an area of 7,000 square miles.

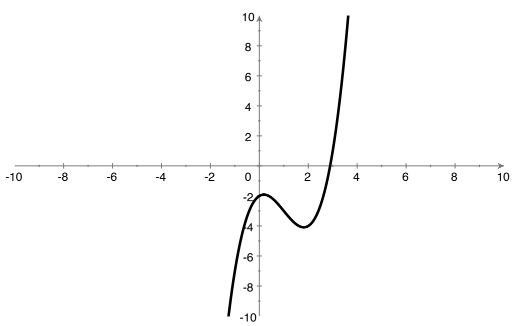
6) We're going to describe the characteristics of various polynomial functions. Here are the terms we will use:

Domain: All possible input values (x values)
Range: All possible output values (y values)
Continuity: A function is continuous if we can draw it without lifting our pencil
Increasing: A function is increasing over an interval if it has a positive slope over that interval
Turning point: A turning point is where a function changes from increasing to decreasing
Concave up: A function is concave up over an interval if its slope is increasing (shaped like a cup)
Concave down: A function is concave down over an interval if its slope is decreasing (shaped like a frown)
Inflection point: A point at which the function changes concavity
Extrema: A local maximum or minimum of a function
End behavior: What happens at the extreme left and right sides of the graph? Do they go in the same direction?
Maximum number of real zeros: How many times could the function cross the x-axis?
Minimum number of real zeros: What's the smallest number of times the function could cross the x-axis?



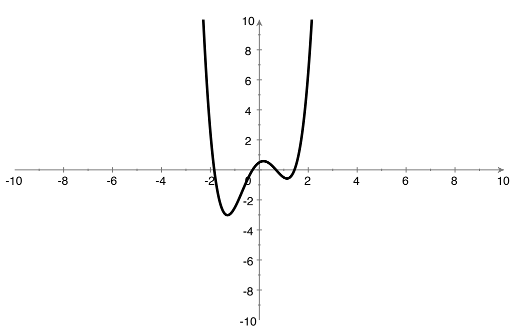
$$f(x) = 2x^2 + 2$$

Degree		Extrema	
Domain		Inflection Points	
Range		End Behavior	
Continuous?		Maximum # of real zeros	
Turning Points		Minimum # of real zeros	



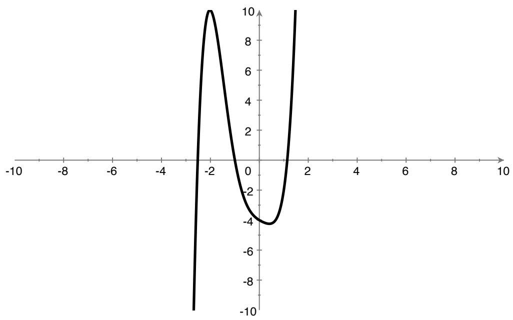
$$f(x) = x^3 - 3x^2 + x - 2$$

Degree		Extrema	
Domain		Inflection Points	
Range		End Behavior	
Continuous?		Maximum # of real zeros	
Turning Points		Minimum # of real zeros	



$$f(x) = x^4 - 3x^3 + x + 0.5$$

Degree		Extrema	
Domain		Inflection Points	
Range		End Behavior	
Continuous?		Maximum # of real zeros	
Turning Points		Minimum # of real zeros	



$$f(x) = x^5 + 2x^4 - x^3 + x^2 - x - 4$$

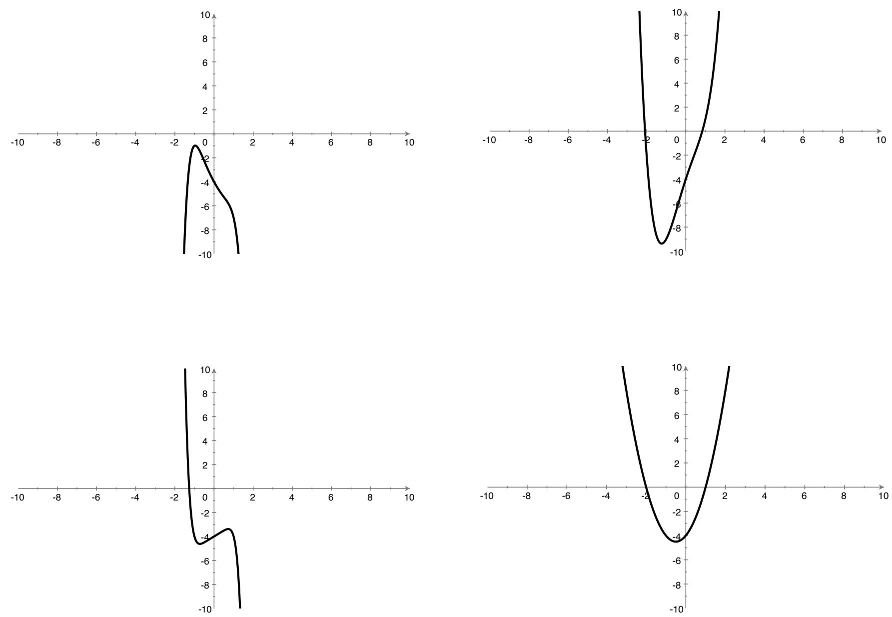
Degree		Extrema	
Domain		Inflection Points	
Range		End Behavior	
Continuous?		Maximum # of real zeros	
Turning Points		Minimum # of real zeros	

7) See if you can generalize the characteristics of polynomial functions based on their degrees:

	Even-degree polynomial (degree n)	Odd-degree polynomial (degree n)
Domain		
Range		
Continuity		
End Behavior		
Maximum number of real zeros		
Minimum number of real zeros		
Maximum number of extrema		
Maximum number of inflection points		
End behavior if the leading coefficient is positive		
End behavior if the leading coefficient is negative		

8) Match each function with its formula. Explain your reasoning.

$$a(x) = x^4 - x^2 + 5x - 4$$
$$b(x) = 3x^3 - x^2 + 2x - 4$$
$$c(x) = -x^6 + x^2 - 3x - 4$$
$$d(x) = -x^7 + x - 4$$



9) Find the real zeros (x-intercepts) of the following polynomial functions.

$$f(x) = x^4 - 11x^3 + 39x^2 - 45x + 2 = 2 + x(x-5)(x-3)(x-3)$$

$$g(x) = x^6 - 3x^4 + 2x^2 \quad \text{(factor)}$$

$$h(x) = (x-2)^2(2x+3)$$

$$i(x) = x^3 + 4x^2 + x - 6 \quad \text{(graphically)}$$

10) We can also construct a polynomial if we're given the degree and its zeros. For example, suppose we want a 3rd-degree polynomial with zeros of X = -4, 0, and 6. Construct the polynomial and graph it to check your answer. Could we construct a 4th-degree polynomial with those same zeros?

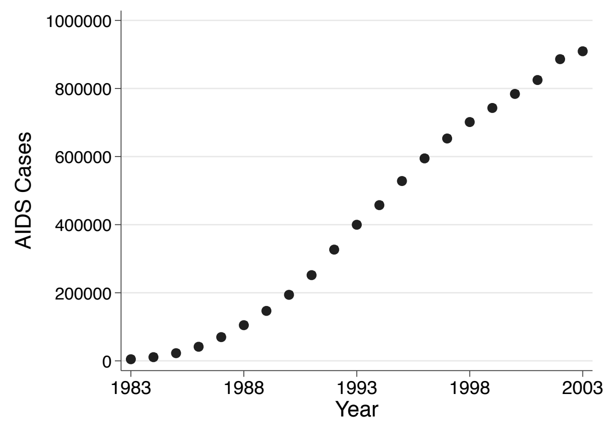
11) Construct a polynomial with zeros $X = 3$ and $X = (2+i)$. Construct a polynomial with zeros $X = 0$, $X = -2$, and $X = (-3-i)$.

12) So far in this class, we've learned how to use least-squares regression to fit linear, exponential, logarithmic, and logistic functions to data. We can use those same methods to fit polynomial functions to data.

The following tables and scatterplot display the accumulated total number of reported cases of AIDS in the U.S. since it was first diagnosed in 1983 through 2003.

Year	AIDS Cases
1983	4,589
1984	10,750
1985	22,399
1986	41,256
1987	69,592
1988	104,644
1989	146,574
1990	193,878
1991	251,638
1992	326,648
1993	399,613

Year	AIDS Cases
1994	457,280
1995	528,144
1996	594,641
1997	653,084
1998	701,353
1999	742,709
2000	783,976
2001	824,809
2002	886,098
2003	909,269



Enter this data into your calculator and verify the scatterplot looks like the one provided above.

a) Find the linear function that best fits this data. Record the formula and the r^2 value. Interpret the slope and y-intercept.

b) Why shouldn't we choose an exponential function to model this data?

c) Find the cubic function that best fits this data (CubicReg). Record the formula and the r^2 value. Which model appears to fit the data better, the linear model or the cubic model?

13) Should we choose that cubic function to model the cumulative number of AIDS cases in the U.S.? Graph this function on your calculator. Then, extend the window on your graph to display the function over the interval $0 \le x \le 35$. Sketch that graph below and comment on how well this function describes the underlying relationship between time and cumulative number of AIDS cases.

13) Use your calculator to find the maximum value on this function. At what year does our model suggest the cumulative number of AIDS cases will be at a maximum? How many cumulative AIDS cases would we have at this time?

According to the CDC (where I got this data), "The cumulative estimated number of AIDS diagnoses through 2010 in the United States and 6 U.S. dependent areas was 1,163,575."

Source: http://www.cdc.gov/hiv/topics/surveillance/basic.htm

13) The following table displays the number of nuclear warheads, worldwide, since 1950. What type of polynomial function would be reasonable to use as a model for the number of nuclear warheads as a function of time?

Find the equation for the polynomial function that most appropriately models this data. Write that equation below and record the r^2 value.

Year	Warheads
1950	374
1960	22,069
1970	38,696
1975	47,604
1980	55,246
1985	64,519
1990	59,239
1995	40,344
2000	32,632
2002	30,425
2004	29,308
2006	26,854

Predict the number of nuclear warheads worldwide in 2010 (assuming this trend continues).

Using your calculator, estimate the location of the turning points in the function. According to this model, what was the maximum number of warheads between 1960 and 2000? When did this maximum occur? What was the minimum number and when did it occur?

Source: Assadourian, E., et al. (2008). *Vital Signs 2007-2008: The Environmental Trends that are Shaping our Future.*