Houghton Mifflin Mathematics

Essential Characteristics of Effective Mathematics Instruction

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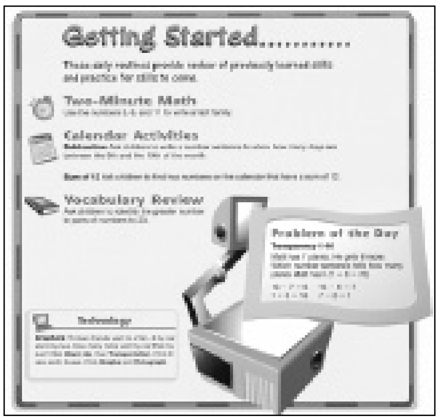
A balanced, comprehensive, and rigorous curriculum is a necessary component for student success in mathematics. A solid curriculum alone, however, is not sufficient to ensure student success. There is increasing evidence that effective instruction is as important as curriculum in helping students learn mathematics (Boaler, 2001; Cohen, Raudenbush, & Ball, 2000; Sanders, 1998). Educators and the public debate the most effective way to teach mathematics; while some emphasize hands-on activities and student discovery, others advocate teacher exposition and paper-and-pencil practice. However, just as a recent report by the National Research Council (Kilpatrick et al., 2001) argues that an effective mathematics curriculum is balanced—focused on conceptual understanding, procedural fluency, and reasoning and problem solving—effective mathematics instruction is similarly balanced.

As the authors of *Principles and Standards for School Mathematics* wrote, "there is no 'right way' to teach mathematics" (NCTM, 2000, p.18). However, we recognize that there are some mathematics teachers who are more effective than others. Effective mathematics teachers, whether they tend toward the student-discovery or the teacher-directed end of the continuum, do certain things in common when delivering mathematics instruction. There are five essential characteristics of effective mathematics lessons: the introduction, development of the concept or skill, guided practice, summary, and independent practice. There are many ways to implement these five characteristics, and specific instructional decisions will vary depending on the needs of the students, the objective being taught, available resources, and teacher preference, but all five characteristics are always present.

The Introduction

Every effective math lesson has a distinct beginning. There are a variety of ways to begin a mathematics lesson, including discussion of a "Problem of the Day," a related calendar activity, mental-math practice, or another daily routine.

However, regardless of how a mathematics lesson starts, two things always happen at the beginning. First, the opening activities are designed to access students' prior knowledge. Second, it is important to share with students the lesson's mathematics objective and the reason for learning it. This helps the students and the teacher focus on what is important and makes the purpose of the selected learning activities clear. An effective mathematics curriculum, therefore, makes the mathematics objectives clear to students, parents, and teachers.



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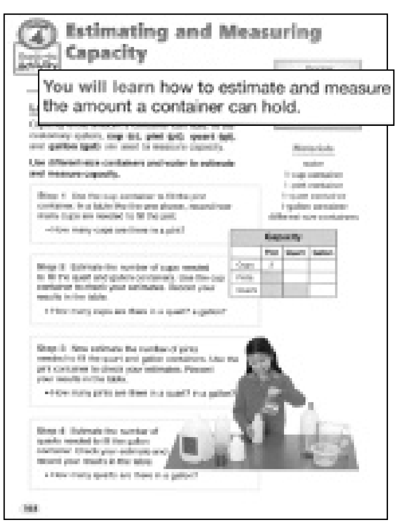
Development of the Concept or Skill

This characteristic of effective lessons focuses on instructional strategies and activities selected by the teacher and designed to actively engage students in learning and to support mastery of the objective. The selection of worthwhile mathematical tasks is critical, since a series of interesting activities alone does not promote student learning. "The teacher's role in choosing worthwhile problems and mathematical tasks is crucial…there are many, many problems that are interesting and fun but that may not lead to the development of the mathematical ideas that are important for a class at a particular time" (NCTM, 2000, p. 53).



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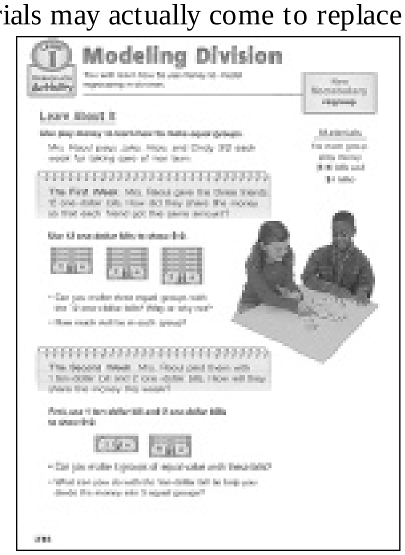
A quality mathematics program will include worthwhile mathematical tasks, and models to assist teachers in making sound instructional decisions that advance student learning. In the last fifteen years, there has been increasing emphasis on the use of concrete materials and activities to promote student learning and understanding of mathematics. The representation-process standard calls on instructional programs to use representations to model and interpret mathematical phenomena (NCTM, 2000). The research is conclusive that students whose teachers emphasize higher-order thinking skills and hands-on activities perform better overall than students whose teachers do not emphasize these skills and activities (Wenglinsky, 2000; Grouws & Smith, 2000).

The use of manipulatives and other hands-on activities alone, however, does not ensure student understanding of mathematics. Students have to do more than simply explore activities; the teacher must help students process activities in a meaningful way. Used inappropriately, the use of concrete materials may actually come to replace a student's thinking and interfere with learning (Fennell and Rowan, 2001). The value of using manipulatives, therefore, depends not on whether they are used, but on how they are used with students (Cawelti, 1999). In order to promote understanding, it is



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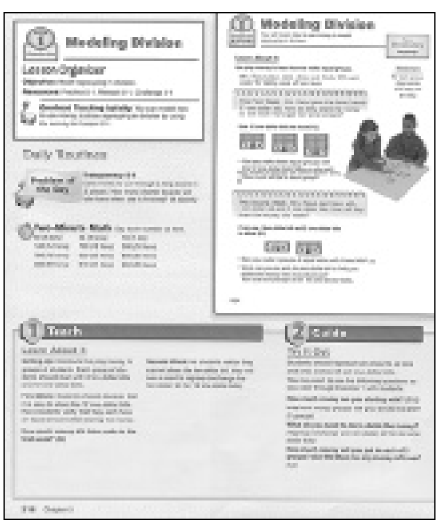
necessary to make connections between concrete representations and the mathematical ideas they represent. As the recent report by the National Research Council points out, “physical materials are not automatically meaningful to students and need to be connected to the situations being modeled” (Kilpatrick et al., 2001, p. 7). An effective teacher mediates students’ understanding of the representations and serves as a bridge between the concrete and the abstract.

An effective mathematics curriculum assists teachers in helping students cross that bridge. For example, in second grade when students are learning the traditional regrouping algorithm for subtraction, each step in the algorithm can be linked to the base-ten materials that represent it, so that students learn the algorithm with understanding. Again, as argued by Kilpatrick et al, procedural fluency and conceptual understanding are interwoven. “Understanding makes learning skills easier...a certain level of skill is required to learn many mathematical concepts with understanding” (Kilpatrick et al., 2001, p. 122). Moreover, there is evidence that when teachers balance and connect the teaching of skills with conceptual understanding, students perform better on state assessments (Knapp, Adelman, et al., 1995).



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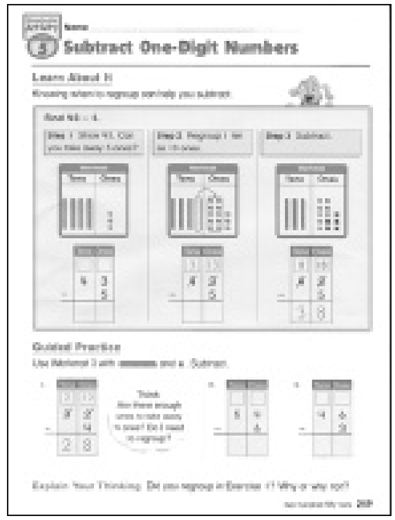

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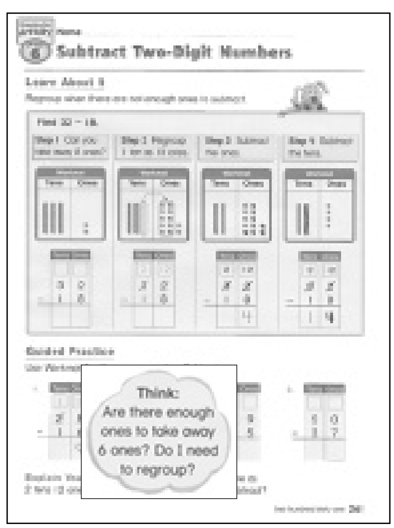
Guided Practice

As the development phase continues, it is necessary for students to move from learning activities to initial practice under the guidance of the teacher. During this phase the teacher gives students certain problems to work on alone or in groups and then these problems are discussed together. This phase serves as a check on student understanding and, depending on the discussion, might indicate a need for additional teaching and exploration of the mathematics.

The value of guided practice in instruction has been recognized for a long time and most textbooks include guided practice in their problem sets. However, an effective mathematics curriculum includes metacognitive questions that students can ask themselves, or that teachers can discuss with students, as they complete these initial practice problems. Without these key questions, guided practice becomes routine skills practice and does not contribute to student learning.



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Summary

The summary component of effective mathematics lessons can be the most important component, particularly when students have been involved in hands-on activities or investigations. Effective teachers think about what questions they can ask to ensure that students understand the mathematics. During this phase of the lesson the teacher, through careful questioning and discussion, helps students connect the mathematical results of the activities to the objective of the lesson, to previously learned mathematics, and to mathematics yet to be learned. These interactions between students and teachers, and between students and students, have a powerful influence on student learning.

Practice

Mathematics instruction that uses manipulatives to facilitate students' conceptual understanding does not suggest that students should not practice (Burrill, 1997). Practice is necessary to master any skill, and mathematics is no exception. The National Research Council Report similarly argues that "significant instructional time [should be] devoted to developing concepts and methods, and carefully directed practice, with feedback, [should be] used to support student learning (Kilpatrick et al., 2001, p.11). Much of this practice will take the form of written practice, but it can also be found in carefully designed games or activities that reinforce concepts and skills.

Effective mathematics instruction, therefore, is not found at either end of the continuum. Rather, it depends on well defined mathematics objectives, worthwhile and engaging mathematical tasks, appropriate use of hands-on activities to build conceptual understanding, quality discussion of the mathematical tasks, and meaningful practice to ensure mastery of those concepts and skills.

References

Boaler, J. 2001. Pedagogy and power: Exploring the relationship between "reform" curriculum and equity. Paper presented at the annual meeting of the American Educational Research Association, Seattle, WA.

Cawelti, Gordon, Ed. *Handbook of Research on Improving Student Achievement*. Arlington, VA: Educational Research Service, 1999.

Cohen, D. K., et al.. *Resources, Instruction, and Research*. Seattle, WA: Center for the Study of Teaching and Policy, University of Washington, 2000.

Fennell, Francis (Skip) and Tom Rowan. 2001. Representation: An important process for teaching and learning mathematics. *Teaching Children Mathematics*, vol 7, no. 5.

Grouws, D. A., and M. S. Smith. NAEP findings on the preparation and practices of mathematics teachers. In E.A. Silver and P.A. Kenney, eds. *Results From the Seventh Mathematics Assessment of the National Assessment of Educational Progress*. Reston, VA: National Council of Teachers of Mathematics, 2000.

Kilpatrick, J., et al, eds. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press, 2001.

Knapp, M.S., et al.. *Teaching for Meaning in High-Poverty Classrooms*. NY: Teachers College Press, 1995.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

Sanders, W.L. (1998) Value-added assessment. The School Administrator, 55(11), 24-27.