Discrete Distributions Quiz:

Complete this after Activity 8. When you show me your work, I will give you the answers.

For each question:
a) Identify the distribution you use to solve the problem
b) Calculate the expected value
c) Answer the question

Situation: 7 boys and 10 girls take a test. Each student has a 20% chance of failing the test.

1. What is the probability that more than 5 students will fail this test?

2. If the students take the test one-after-another, what is the probability that the first failure will come from the second student? What's the probability that the first failure will be the fifth student? What's the probability that the first failure will come <u>before</u> the 6th student?

3. If the students take the test one-after-another, what is the probability that the third failure will come from the tenth student?

4. Suppose three students fail the exam. What is the probability that at least two of the failures were boys? Do not calculate the expected value for this question.

5. Suppose a state experiences an average of three tornados every six months. If the number of tornados observed in any *one-year period* follows a Poisson distribution, calculate the probability of observing between 5 and 7 tornados in a year.

Answers:

1. What is the probability that more than 5 students will fail this test?

   Binomial Distribution with 17 trials and p = 0.2.

   $$E[x] = np = (17)(.2) = 3.4$$

   $$P(X > 5) = 1 - P(X \le 5) = 1 - binomcdf(17,.2,5) = 1 - .894 = 0.106$$

2. If the students take the test one-after-another, what is the probability that the first failure will come from the second student? What's the probability that the first failure will be the fifth student? What's the probability that the first failure will come before the 6th student?

   We are looking for the first failure in a series of trials, so this is a geometric distribution problem.

   $$E[x] = \frac{1}{p} = \frac{1}{.2} = 5$$

   $$P(\text{1st failure from 2nd student}) = P(X = 2) = (.8)^1 (.2)^1 = .16 = geompdf(.2,2)$$

   $$P(\text{1st failure from 5th student}) = P(X = 5) = (.8)^4 (.2)^1 = .08192 = geompdf(.2,5)$$

   $$P(\text{1st failure before 6th student}) = P(X < 6) = P(X \le 5) = geomcdf(.2,5) = 0.67232$$

3. If the students take the test one-after-another, what is the probability that the third failure will come from the tenth student?

   We are looking for the rth instance, so this is a negative binomial distribution.

   $$E[x] = \frac{r}{p} = \frac{3}{.2} = 15$$

   $$P(\text{3rd failure from 10th student}) = P(X = 10) = \binom{10-1}{3-1} (.2)^3 (.8)^7 = 0.0604$$

4. Suppose three students fail the exam. What is the probability that at least two of the failures were boys?

We have two populations – boys and girls – so we must use the hypergeometric distribution.

$$P(\text{at least 2 boys}) = P(X \geq 2) = \left[P(X=2) + P(X=3)\right]$$

$$P(X=2) = \frac{\binom{7}{2}\binom{10}{1}}{\binom{17}{3}} = 0.3088 \qquad P(X=3) = \frac{\binom{7}{3}\binom{10}{0}}{\binom{17}{3}} = 0.0515$$

$$P(X \geq 2) = 0.3088 + 0.0515 = 0.3603$$

5. Suppose a state experiences an average of three tornados every six months. If the number of tornados observed in any *one-year period* follows a Poisson distribution, calculate the probability of observing between 5 and 7 tornados in a year.

Since the problem says it follows a Poisson distribution, we should probably use the Poisson distribution.

We are interested in a 1-year block of time, so we need to find the expected number of tornados in that 1-year period. Since we expect 3 tornados in 6 months, we expect 6 tornados in any year.

Therefore, $E[x] = \lambda = 6$

$$P(5 \leq X \leq 7) = P(X=5) + P(X=6) + P(X=7)$$

$$P(X=5) = \frac{e^{-6}(6)^5}{5!} = 0.1606$$

$$P(X=6) = \frac{e^{-6}(6)^6}{6!} = 0.1606$$

$$P(X=7) = \frac{e^{-6}(6)^7}{7!} = 0.1377$$

Therefore, $P(5 \leq X \leq 7) = 0.4589$