Assignment #2: Introduction to Probability & Counting Name: ______________________________

Note: I fully intend you to answer the following three "tricky problems" incorrectly. You could look up the answers online (probably), but go ahead and write the answers that you think are correct. We'll discuss these questions in class (and I won't ask any like questions like these on a test in this class).

Tricky Problem A: Amy has two children. You know Amy's youngest child is a female.
Betty has two children. You know one child is female; you know nothing about the other child.

What's the probability that Amy has two female children? ______________

What's the probability that Betty has two female children? ______________

Tricky Problem B: Chris tells you, "I have two children. One is a boy born on a Tuesday." He does not tell you anything about the other child.

What's the probability that Chris has two boys? ______________

Tricky Problem C: You are a contestant on the *Let's Make a Deal* game show. You see 3 doors before you. You'll get to choose one door and win whatever prize is behind that door. The game show host tells you that a car is behind one door and goats are behind the two other doors.

You choose one door and the host, who knows what's behind all three doors, opens a door you didn't choose to reveal a goat. He then gives you a choice: you can stay with the door you originally selected or you can switch to the other closed door. Should you stick with your original choice or go with the other door?

What's the probability the car is behind the door you original chose? ______________

What's the probability the car is behind the other door? ______________

Ok, now let's try some questions that you should be able to answer.

1. Briefly explain how you could use a relative frequency approach to estimate the probability of passing MATH 300.

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2. As of September 2012, there were 105 million registered .com web domains registered (including google.com and bradthiessen.com).

http://www.enterprisenetworkingplanet.com/netsp/dot-com-domains-top-105-million-names.html

How many domain names consisting of just three (of the 26 English) letters in sequence can be formed?

Answer: ________________________________

How many 3-character domain names could be formed if both letters and numbers are allowed?

Answer: ________________________________

How many 3-character domain names could be formed if both letters and numbers are allowed but the same character cannot be used more than once?

Answer: ________________________________

3. I need to develop a test for this class. Suppose I've written 8 multiple-choice questions, 10 true/false questions, and 12 short-answer questions.

Let's suppose the order of the questions doesn't matter. How many different 6-question tests could I create by selecting among the 30 questions I've developed?

Answer: ________________________________

This time, suppose the order of the questions does matter. How many different 6-question tests could I create by selecting among the 30 questions I've developed?

Answer: ________________________________

Suppose I choose 6 questions at random. How many ways are there to choose two questions of each type (i.e., 2 multiple-choice, 2 true/false, and 2 short-answer questions)?

Answer: ________________________________

Suppose I choose 6 questions at random. What's the probability I choose two questions of each type (i.e., 2 multiple-choice, 2 true/false, and 2 short-answer questions)?

Answer: ________________________________

4. Suppose a little league baseball team has 15 players on its roster.

   How many different 9-player starting line-ups could be formed from these 15 players?

   Answer: ________________________________

   Suppose the coach has selected the 9 players who will start the game. The coach now needs to decide the batting order (who will bat first, second, third, …). How many different batting orders are possible?

   Answer: ________________________________

   Suppose it's a new day and the coach needs to choose 9 players to start the game <u>and</u> a batting order for those 9 players. What's the total number of starting batting orders the coach could form?

   Answer: ________________________________

   Suppose 5 of the 15 players are left-handed. The coach needs to choose 3 outfielders and 6 players for other positions. How many ways are there to choose 3 left-handed players for the outfield and 6 right-handed players for all the other positions?

   Answer: ________________________________

Suppose you only have 50 cents in your pocket and you want to buy an ice cream cone. The owner of the ice cream shop offers a random price determined as follows: **You roll a pair of fair 6-sided dice and the price is the larger number followed by the smaller number (in cents).**

Source: Allan Rossman: http://statweb.calpoly.edu/arossman/stat325/notes.html

5. Let's use a computer simulation to estimate the probability of affording the ice cream.

   Using a computer program called *R*, we could simulate this scenario using the following code:

```
Code: d1 = sample(1:6,10000, replace=TRUE)
d2 = sample(1:6,10000, replace=TRUE)
price = 10*pmax(d1,d2) + pmin(d1,d2)
afford=(price<=50)
sum(afford)
```

   Regardless of your background in programming, let's see if we can make sense of that code.

   1. The first line records 10,000 random rolls of a 6-sided die. To do this, the code tells the computer to randomly sample 10,000 values (with replacement) from the integers 1-6. These values are recorded in a variable (think of it as a column in a spreadsheet) called d1.
   2. The second line uses similar logic to simulate 10,000 rolls of our second 6-sided die. This variable is called d2.
   3. The 3rd line calculates the "price" we rolled in each of the 10,000 trials. To do this, it takes:
      10 x (the value of the die with the bigger number) + (the value of the die with the smaller number)
   4. The 4th line determines if we could have afforded the ice cream cone in each of our 10,000 trials. To do this, a variable named *afford* is created. We can afford the ice cream cone if, on any given trial, our *price* < 50.
   5. The last line sums the number of times we could afford the ice cream (out of our 10,000 trials).

   A) I ran this code in R and received the following output: sum(afford) = 4419. Based on this simulation, what's your best estimate of the probability that we could afford the ice cream cone?

      Answer: ____________________

   B) I ran this code 3 more times and received the following output: sum(afford) = 4361, sum(afford) = 4592, sum(afford) = 4461. Based on this additional information, what's your best estimate of the probability we could afford the ice cream?

      Answer: ____________________

6. Now let's try to estimate this probability using an exact enumeration method.

   A) List all the possible outcomes from rolling a pair of fair, 6-sided dice. How many possible outcomes are there?

      Total number of outcomes: ____________________

B) Looking at the sample space you just listed, circle the outcomes that represent the event that you can afford the ice cream. How many outcomes did you circle? From this, determine the (exact) probability that you can afford the ice cream cone?

Check your answer by comparing it to the estimates from the simulation method.

Probability of affording the ice cream: ____________________

C) Suppose you are given a similar problem. Which method would you prefer to use to get probabilities: the simulation method or the exact calculation? Why? Identify a potential strength and weakness for each method?

7. I once served as a magician's assistant at Playland Not-at-the-Beach in El Cerrito, CA: http://www.playland-not-at-the-beach.org/ Based on this experience, I've come up with the following (lame) magic trick:

   I ask an audience member to think of a 2-digit number, both digits must be unique odd numbers. I then guess his or her number and wait for applause.

   Ok, so it's not the best trick in the world, but suppose I actually did this trick. In fact, suppose I had each of 1,000 audience members think of a 2-digit number (both digits must be unique odd numbers). Further suppose that when I stated my guess, 350 audience members admitted that I had guessed correctly.

   A) If I had no magical abilities (and just guessed a number at random), what is the probability that I would have correctly guessed an individual's randomly-chosen number? How many audience members out of 1000 would have chosen that number at random?

   Probability: ____________________ Number of audience members: ____________________

   B) I guessed 350 correctly out of 1000. Does this provide evidence that I have magical abilities? Identify some other plausible explanations for how I was able to guess the number for 350 audience members.

   C) Assuming every audience member chose a number at random and I guessed a number at random, what's the likelihood that I would have guessed 350 or more correctly out of 1000? Go ahead and guess, since you don't know how to calculate this yet.