Activity 2a: Tricky Probability Problem Solutions

A) Amy has two children. You know Amy's youngest child is a female.
Betty has two children. You know one child is female; you know nothing about the other child.
What's the probability that Amy has two female children?
What's the probability that Betty has two female children?

With two children, the sample space is: BB, BG, GB, GG (where the youngest child is listed first)

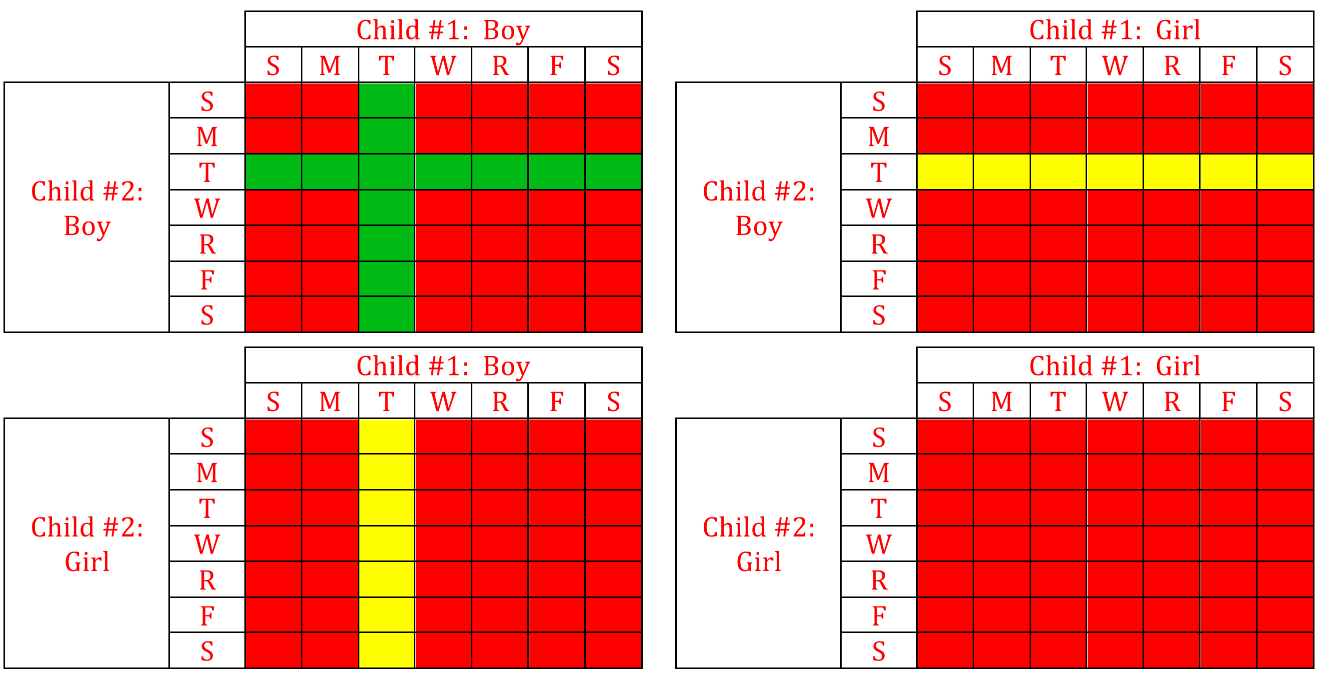
Since Amy's youngest child is female, her sample space is: GB or GG. Therefore, the probability that Amy has two female children is ½ = 0.50.

Since we know Betty has one female child, her sample space is: GB, BG, or GG. Therefore, the probability that Betty has two female children is 1/3 = 0.333.

B) Chris tells you, "I have two children. One is a boy born on a Tuesday." He does not tell you anything about the other child. What's the probability that Chris has two boys?

Let's visualize the sample space. If we only knew Chris had 2 children, we would have 4 possibilities for gender (BB, BG, GB, GG) and 49 possibilities for the day of the week each child was born (7 possible days for the first child x 7 possible days for the second child).

Since we know at least one child was born on Tuesday, we can eliminate all the possibilities in which a boy was not born on Tuesday. Since we know one child is a boy, we can eliminate all the possibilities with two female children. I've eliminated these possibilities by shading them red in the display below:



The yellow/green boxes represent all 27 possibilities (with our given information). The 13 green boxes represent what we're looking for – two boys.

So the probability that Chris has two boys is 13/27 = 0.48.

C) Suppose you're on the *Let's Make a Deal* game show. You see 3 doors before you: behind one door is a car; behind the other two doors are goats. You choose one door (Door #1) and the host, who knows what's behind all 3 doors, opens another door (Door #2) to shows a goat behind it. He then gives you a choice – you can stay with Door #1 or you can switch to Door #3. Is it to your advantage to switch to Door #3? What's the probability you win the car if you stay with your first choice? What's the probability you win the car if you switch to the other door?

Suppose you first choose Door #1. The following table lists all possibilities:

	Door #1	Door #2	Door #3
Possibility A	**Car**	Goat	Goat
Possibility B	**Goat**	Car	Goat
Possibility C	**Goat**	Goat	Car

This table lists all possible outcomes:

	If you switch doors	If you stay with Door #1
Possibility A	Goat	Car
Possibility B	Car	Goat
Possibility C	Car	Goat

Therefore, switching doors gives you a 2/3 chance of winning.

Another explanation: A player who stays with the first door chosen can only win if that door has a car. Therefore, a player has a 1/3 chance of winning with the first chosen door and a 2/3 chance of winning by switching doors.

D) Suppose we have 25 students in this class. What's the probability of finding two students in this class with the same birthday (month and day)? How large would the class need to be to give us a 99% chance of finding two students with the same birthday?

Let's agree to ignore leap year (February 29th) birthdays – that way, we can work with the fact that a year is 365 days long. Let's also assume that people have an equal chance of being born on any of the 365 days.

What's the probability that <u>two</u> people share a birthday?

The first person can have any birthday, so that gives this person 365 possible birthdays out of 365 days total. So the probability of the first person having a specific birthday is 365/365 = 1.0.

The second person has a 1/365 chance of having the same birthday as the first person. When events (like individual's birthdays) are independent of each other, the probability of all the events occurring (both individuals having the same birthday) is equal to a product of the probabilities of each event occurring. Therefore, the probability of both individuals having the same birthday is: (365/365) x (1/365) = 1/365.

What's the probability that <u>three</u> people share a birthday?

The probability of the first two people having the same birthday is still 1/365. Likewise, the probability of the first and third individuals sharing a birthday is 1/365. But what's the probability that the second and third person share a birthday? And what's the probability that they all 3 share the same birthday?

An easier way to calculate the probability of *at least two people sharing a birthday* is to calculate its complement:

P(at least two people share a birthday) = 1 – P(no two people share a birthday)

The first person can be born any day. We want to calculate the probability that no two people share a birthday, so the second person can be born on any of 364 days. Likewise, the third person can be born on any one of 363 days.

To find the probability that the first, second, and third person have different birthdays, we can multiply:

(365/365) x (364/365) x (363/365) = 132,132 / 133,225 = 0.9918 = 99% chance.

Therefore, the probability that at least two do share a birthday is 1%.

To find the probability that <u>four</u> people have different birthdays, we can multiply:

(365/365) x (364/365) x (363/365) x (362/365) = 0.9836 = 98% chance

Therefore, the probability that at least two do share a birthday is 2%.

We could keep going. The probability that no two people in a class of 25 share a birthday would be:

(365/365) x (364/365) x … x (341/365) = 43% chance

Therefore, the probability that at least two do share a birthday is 57%.

So we've answered our first question. There is a 57% chance that we'll find at least two students sharing a birthday in a class of 25.

The formula for the probability of finding no shared birthdays in a group of **n** people is:

$$\frac{365}{365} * \frac{364}{365} * \frac{363}{365} * \ldots * \frac{365-n+1}{365} = \frac{365 * 364 * 363 * \ldots * (365-n+1)}{365^n}$$

Using permutations, we can rewrite the formula as:

$$\frac{{}_{365}P_n}{365^n}$$

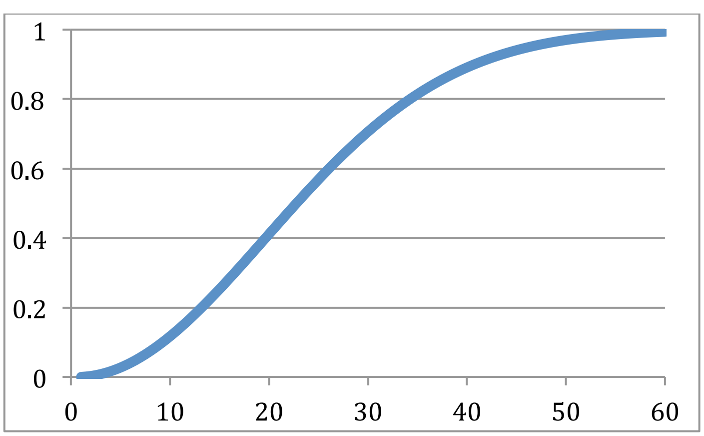
Which we could also rewrite as:

$$\frac{\dfrac{365!}{(365-n)!}}{365^n}$$

Let's verify this formula gives us the correct answer for a class of 25 students:

$$\frac{365!}{(365-n)! * 365^n} = \frac{365!}{(365-25)! * 365^{25}} = \frac{365!}{(340)! * 365^{25}} = \frac{365 * 364 * \ldots * 341}{365^{25}} = 0.4313$$

To find the class size needed for a 99% chance of finding two people with the same birthday, I just calculated the probability for all possible class sizes up to 60. On the chart below, the horizontal axis represents class size and the vertical axis represents the probability that at least two people share a birthday:



Looking at the graph, we get close to 99% once we get past a class size of 50 students.

It turns out that in a group of 57 people, there is a 99% chance that at least two people share a birthday.