Activity #18: Magnitude Scaling & Multidimensional Scaling

Resources: Kruskal, J. & Wish, M. (1978). *Multdimensional Scaling*. Sage Publications.
Nations.sav, fruit.sav, cities.sav

Scaling refers to the process by which we assign numbers corresponding to varying levels of a trait. Some types of scales, such as those found in psychophysics, are easy to create. For example, we typically measure individuals' heights in feet and inches (higher numbers correspond to greater levels of height). ACT scores are scaled from 1-36, with higher numbers corresponding to higher levels of achievement.

Other types of scales, such as those in psychological domains, are more difficult to create. For example, suppose you wanted to scale the severity of a list of crimes. You present subjects with 30 short stories describing various types of crimes (simple burglary to arson to murder). You want to create a "severity of crime" scale such that higher numbers correspond to more serious crimes. How would you go about creating this scale? How many dimensions does this scale have? What would be the range of this scale?

These questions have been raised and addressed by statisticians working in the field of scaling. The purpose of this activity is to familiarize you with some scaling methods. You will not learn the specifics of these scaling methods (the mathematics can be somewhat involved), but you will get an appreciation for what scaling can do.

1. Let's begin by examining how scaling can answer one of life's great mysteries. *Who is the funniest character on The Simpsons?* How would you gather data to answer this question? What would be the limitations of this approach?

2. Through a process called *magnitude scaling*, we can create a ratio scale for humor levels. Here's how we would collect the data:

    a. We administer a survey to a large group of subjects. This survey appears on the next two pages.

    b. We create a standardized stimulus. To do this, we choose a stimulus (character) and assign it a scale value. In this example, I assigned Bart a humor level of 50.

    c. Instruct subjects to rate the humor level of each character compared to the standardized stimulus. For example, if the subject believes a character is 3 times as funny as Bart, the subject should assign that character a value of 150. If the subject believes the character is one-seventh as funny as Bart, the subject should assign that character a value of (50 / 7) = 7.14. Using this method, characters can receive ratings ranging from 0 (not at all humorous) to values approaching infinity (for extremely humorous characters).

    d. Gather the ratings from each subject and calculate the geometric mean rating for each character. To calculate the geometric mean, we do the following:
        i. Take the logarithm of each rating from each subject
        ii. Calculate the average of those log values
        iii. Raise 10 to the power of that average log value.

    e. These geometric means represent the magnitude scaling values of the stimuli. These scale values are on a true ratio scale, we can compare the level of humor for characters through ratios. The final results from this scaling process are displayed on the page following the survey.

Magnitude scaling was developed from the *Power Law* (sometimes called *Steven's Law*). I.E. students may learn about this law in an aesthetics class. If you want to learn more about magnitude scaling, I wrote an easy-to-understand paper about the subject. It really is a powerful form of unidimensional scaling that is underutilized in practice.

3. Let's look at another example of magnitude scaling. This time, I'm interested in learning about the perceived hostility nations have towards the United States. Instead of having subjects simply assign numbers to each nation, I'm going to use another *response modality*.

   I present subjects with a list of 20 nations. I instruct subjects that they will draw lines corresponding to the perceived hostility each nation has towards the U.S. To get them started, I assign a line length to France.

   France

   I then present the subjects with the following survey (only 14 of the 20 nations are displayed here)

Section 5: In this section, you will draw lines so that the length of those lines corresponds to the relative level of hostility each country has towards the U.S. The first line shows the level of hostility France has towards the U.S. If you believe a country has 4.7 times more hostility towards the U.S., you should draw a line 4.7 times as long as the line drawn for France. If you believe a country has less hostility towards the U.S. than France has, you should draw a shorter line. If you believe a country has no hostility towards the U.S., write ZERO instead of drawing a line. This section is two pages long.

France

Afghanistan

Australia

Bosnia

Canada

China

Cuba

Germany

Great Britain

India

Iraq

Israel

Japan

Mexico

To create the magnitude scale, I measured the length of each line from each subject and calculated the geometric mean for each nation across all subjects. These geometric means represent the scale scores for each nation. The results of this survey are displayed below. Once again, the scale values represent a ratio level of measurement.

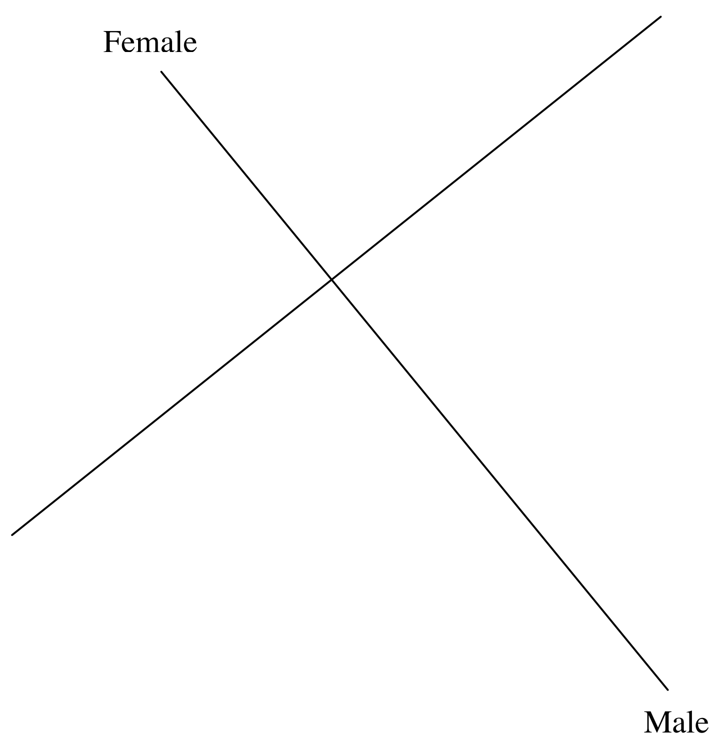
How hostile is each country towards the United States?

Country	Hostility
Iraq	279.16
Afghanistan	214.96
Saudi Arabia	99.73
Pakistan	99.49
Cuba	94.28
China	89.70
Bosnia	81.41
Russia	65.36
Israel	59.00
Germany	57.45

Country	Hostility
Germany	57.45
France	50.00
Japan	48.52
India	44.24
Panama	34.43
Spain	30.48
Mexico	29.13
Canada	18.08
Great Britain	16.35
Sweden	10.31
Australia	6.30

1. Iraq is perceived to have 5.6 times as much hostility towards the U.S. as France and 1.3 times as much hostility as Afghanistan
2. As expected, middle-eastern nations were perceived to be the most hostile
3. The ratings for India had the greatest variation, indicating subjects did not agree on India's level of hostility towards the U.S.
4. Spain received unusually high ratings from several respondents (survey was administered just after the terrorist attack on Spain).
5. English speaking nations had the lowest perceived levels of hostility

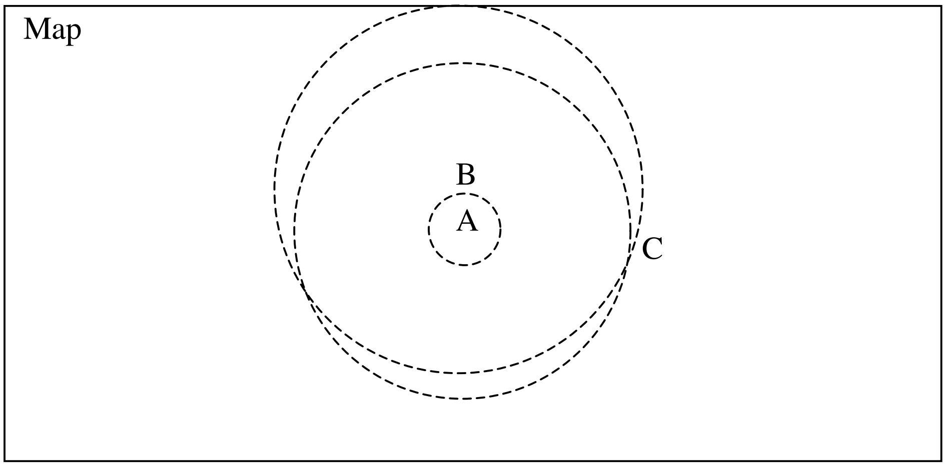
We've been making the assumption that our underlying scale is unidimensional. That is, we assumed that the concept of humor is one-dimensional and that hostility is represented by a single dimension. Suppose, though, that people rate Simpson's characters on the basis of more than one dimension. We can use a process called *multidimensional scaling* to find other dimensions underlying subjects' concept of humor.

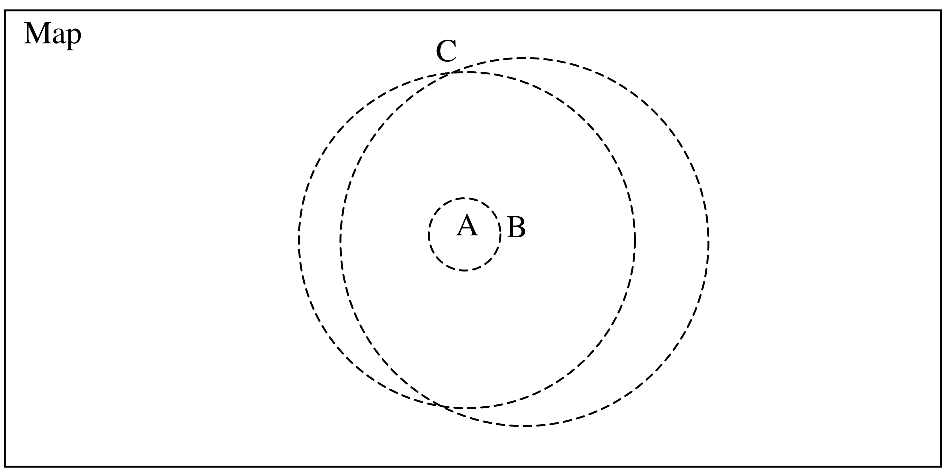


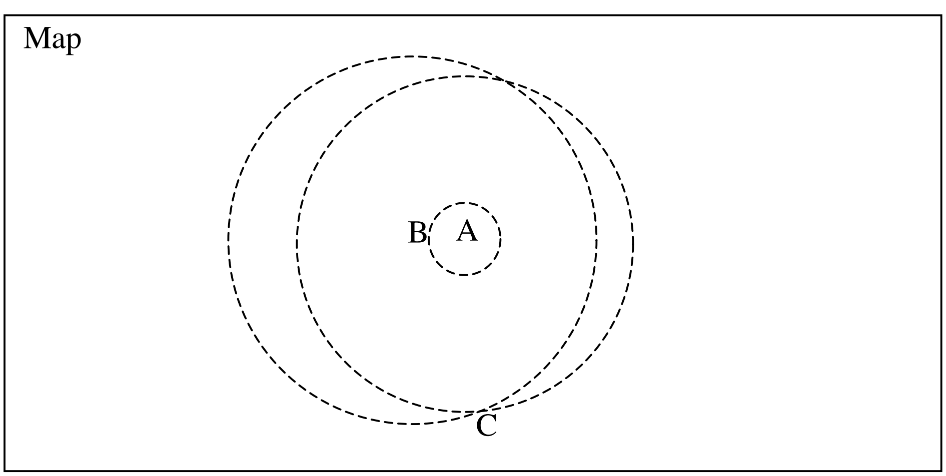
Nonmetric Multidimensional Scaling

4. Suppose I gave you a list of three cities and the distances between them (measured in miles). Without knowing the names of the cities, would you be able to place them on a map? Try it with the following three cities:

	City A	City B	City C
City A	0	53	600
City B	53	0	630
City C	600	630	0







If we begin by placing City A in the center of the map, we can use the following steps to locate the other cities:

1. Using a compass, we draw a circle with a radius of 53 centered at City A. We know City B is located somewhere on the edge of this circle. We arbitrarily place City B on the outside edge of the circle.

2. Since City C is located 600 miles from City A, we draw a circle with a radius of 600 centered at City A.

3. Since City B is located 630 miles from City C, we draw a circle with radius of 630 centered at City B.

4. City C is located at the point of intersection of the two circles just drawn.

This process, although time consuming, would work with a larger number of cities. What are the problems with this method?

Nonmetric multidimensional scaling (MDS) was developed in order to uncover the hidden relationships among data. If used correctly, it can improve our understanding of data by creating a map.

We can use MDS whenever we have data that represent distances. For example:

1. A list of cities and the distances between them.

2. We could take student scores from the ITBS and calculate correlations among the subtests. We would probably find that math and science have a high correlation, whereas math computation and reading may have a low correlation. We could consider these correlations to be “distances” (high correlations represent short distances between variables).

3. We could ask consumers to rate 16 fruits on the basis of how much they enjoyed the taste of each fruit (on a scale from 1-10 – or we could use a magnitude scale). We then calculate correlations among each pair of fruits and use those correlations as distances.

4. We could have consumers rate how well they like 30 brands of breakfast cereal. After calculating correlations and turning them into distances, we could see what factors influence a consumer’s preference for cereal.

Nonmetric MDS was used often in the 1970s by marketing firms. Let’s look at the output from a MDS analysis for a couple examples.

5. **The following table displays the distances between pairs of 10 major U.S. cities. The data were entered into Stata and a nonmetric multidimensional scaling analysis was conducted. The graph shows the configuration of the cities discovered by the analysis.**

- *matrix d = (data entered here)*
- *global names atl chi den hou la mi ny sf sea dc*
- *matrix rownames d = $names*
- *matrix colnames d = $names*
- *matrix list d*

	atl	chi	den	hou	la	mi	ny	sf	sea	dc
atl	0									
chi	587	0								
den	1212	920	0							
hou	701	940	879	0						
la	1936	1745	831	1374	0					
mi	604	1188	1726	968	2339	0				
ny	748	713	1631	1420	2451	1092	0			
sf	2139	1858	949	1645	347	2594	2571	0		
sea	2182	1737	1021	1891	959	2734	2408	678	0	
dc	543	597	1494	1220	2300	923	205	2442	2329	0

- *mdsmat d, names($names)*

The table below shows the proportion of variance among our cities accounted for by 1, 2, 3, 4, 5, and 6 dimensions. Since the cities are on a 2-dimensional map, 2 dimensions account for all the variance.

```
Eigenvalues > 0       =          6          Mardia fit measure 1 =    0.9954
Retained dimensions   =          2          Mardia fit measure 2 =    1.0000
--------------------------------------------------------------------------
             |                   abs(eigenvalue)          (eigenvalue)^2
  Dimension  |  Eigenvalue      Percent    Cumul.        Percent    Cumul.
-------------+------------------------------------------------------------
          1  |   9582144.3        84.64     84.64          96.99     96.99
          2  |   1686820.2        14.90     99.54           3.01    100.00
-------------+------------------------------------------------------------
          3  |   8157.2984         0.07     99.61           0.00    100.00
          4  |   1432.8699         0.01     99.63           0.00    100.00
          5  |   508.66869         0.00     99.63           0.00    100.00
          6  |   25.143486         0.00     99.63           0.00    100.00
--------------------------------------------------------------------------
```

You can see the map is correct except the axes must be rotated

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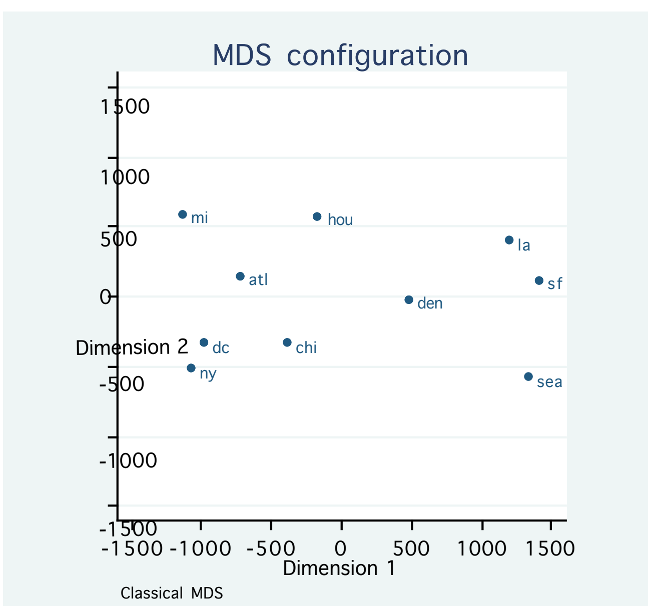
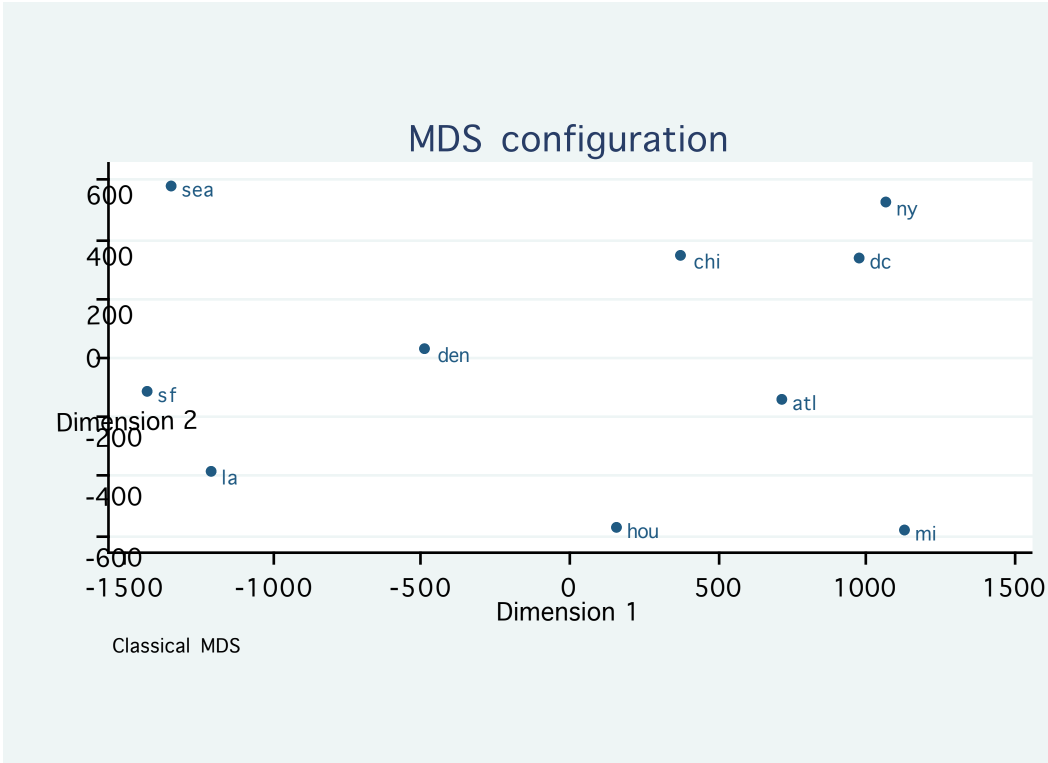


If we flip the horizontal and vertical axes, we get the correct configuration:



6. **The following table displays the data gathered about 25 breakfast cereals.**

brand	cal	protein	fat	Na	fiber	carbs	sugar	K
Cheerios	110	6	2	290	2	17	1	105
Cocoa Puffs	110	1	1	180	0	12	13	55
Honey Nut Cheerios	110	3	1	250	1.5	11.5	10	90
Kix	110	2	1	260	0	21	3	40
Lucky Charms	110	2	1	180	0	12	12	55
Oatmeal Raisin Crsp	130	3	2	170	1.5	13.5	10	120
Raisin Nut Bran	100	3	2	140	2.5	10.5	8	140
Total Corn Flakes	110	2	1	200	0	21	3	35
Total Raisin Bran	140	3	1	190	4	15	14	230
Trix	110	1	1	140	0	13	12	25
Wheaties Honey Gold	110	2	1	200	1	16	8	60
All-Bran	70	4	1	260	9	7	5	320
Apple Jacks	110	2	0	125	1	11	14	30
Corn Flakes	100	2	0	290	1	21	2	35
Corn Pops	110	1	0	90	1	13	12	20
Mueslix Crispy Blnd	160	3	2	150	3	17	13	160
Nut & Honey Crunch	120	2	1	190	0	15	9	40
NG Almond Raisin	140	3	2	220	3	21	7	130
Nutri Grain Wheat	90	3	0	170	3	18	2	90
Product 19	100	3	0	320	1	20	3	45
Raisin Bran	120	3	1	210	5	14	12	240
Rice Krispies	110	2	0	290	0	22	3	35
Special K	110	6	0	230	1	16	3	55
Life	100	4	2	150	2	12	6	95
Puffed Rice	50	1	0	0	0	13	0	15

A multidimensional scaling analysis was conducted to find the “distances” among the cereal brands and to determine how many dimensions account for these “distances.”

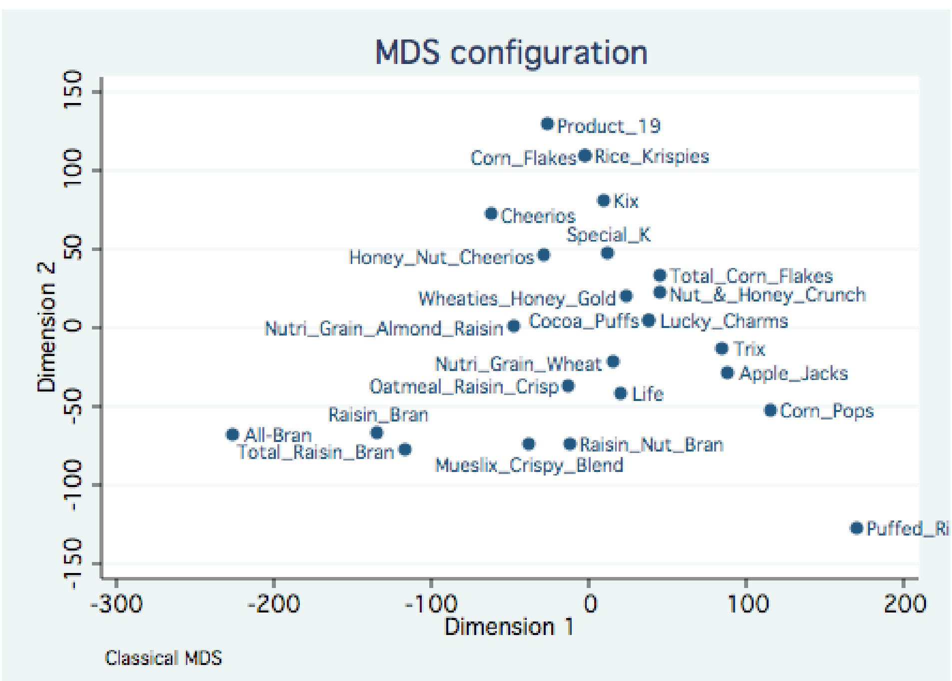
Again, it looks like 2 dimensions account for all the variance in the data.

Eigenvalues > 0 = 8　Mardia fit measure 1 = 0.9603
Retained dimensions = 2　Mardia fit measure 2 = 0.9970

Dimension	Eigenvalue	Percent	Cumul.	Percent	Cumul.
1	158437.92	56.95	56.95	67.78	67.78
2	108728.77	39.08	96.03	31.92	99.70
3	10562.645	3.80	99.83	0.30	100.00
4	382.67849	0.14	99.97	0.00	100.00

Here are the coordinates for each cereal brand on our two dimensions.

brand	dim1	dim2
Cheerios	-61.8271	-72.5534
Cocoa_Puffs	38.5094	-5.1037
Honey_Nut_~s	-28.0515	-46.0667
Kix	9.1693	-81.4942
Lucky_Charms	38.5024	-5.1356
Oatmeal_Ra~p	-12.5635	37.0897
Raisin_Nut~n	-12.0040	73.7800
Total_Corn~s	44.9827	-33.2502
Total_Rais~n	-117.0067	77.9962
Trix	85.0033	12.9330
Wheaties_H~d	23.7367	-19.7182
All-Bran	-226.1791	67.6752
Apple_Jacks	88.6199	28.4323
Corn_Flakes	-1.8069	-109.3770
Corn_Pops	115.5366	52.7072
Mueslix_Cr~d	-37.7449	74.4727
Nut_&_Hone~h	45.3886	-21.9393
Nutri_Grai~n	-47.9441	-0.6082
Nutri_Grai~t	15.2261	21.7290
Product_19	-26.0875	-129.4798
Raisin_Bran	-134.8587	66.7255
Rice_Krisp~s	-2.3710	-109.6115
Special_K	12.1670	-47.9540
Life	20.9036	41.4515
Puffed_Rice	170.6994	127.2995



7. Consumers were presented with samples of 16 fruits. The consumers were asked to rate how well they liked each fruit on a scale of 1-100. Consumers were not told what factors should influence their judgments (taste, appearance, cost, etc).

Ratings given by two subjects to each fruit are displayed below (only 8 of the 15 fruits are displayed).

	Pineapple	Coconut	Strawberry	Banana	Plum	Grapes	Blueberry	Peach
Subject #1	64	47	80	25	16	54	8	78
Subject #2	100	20	75	68	11	50	60	90

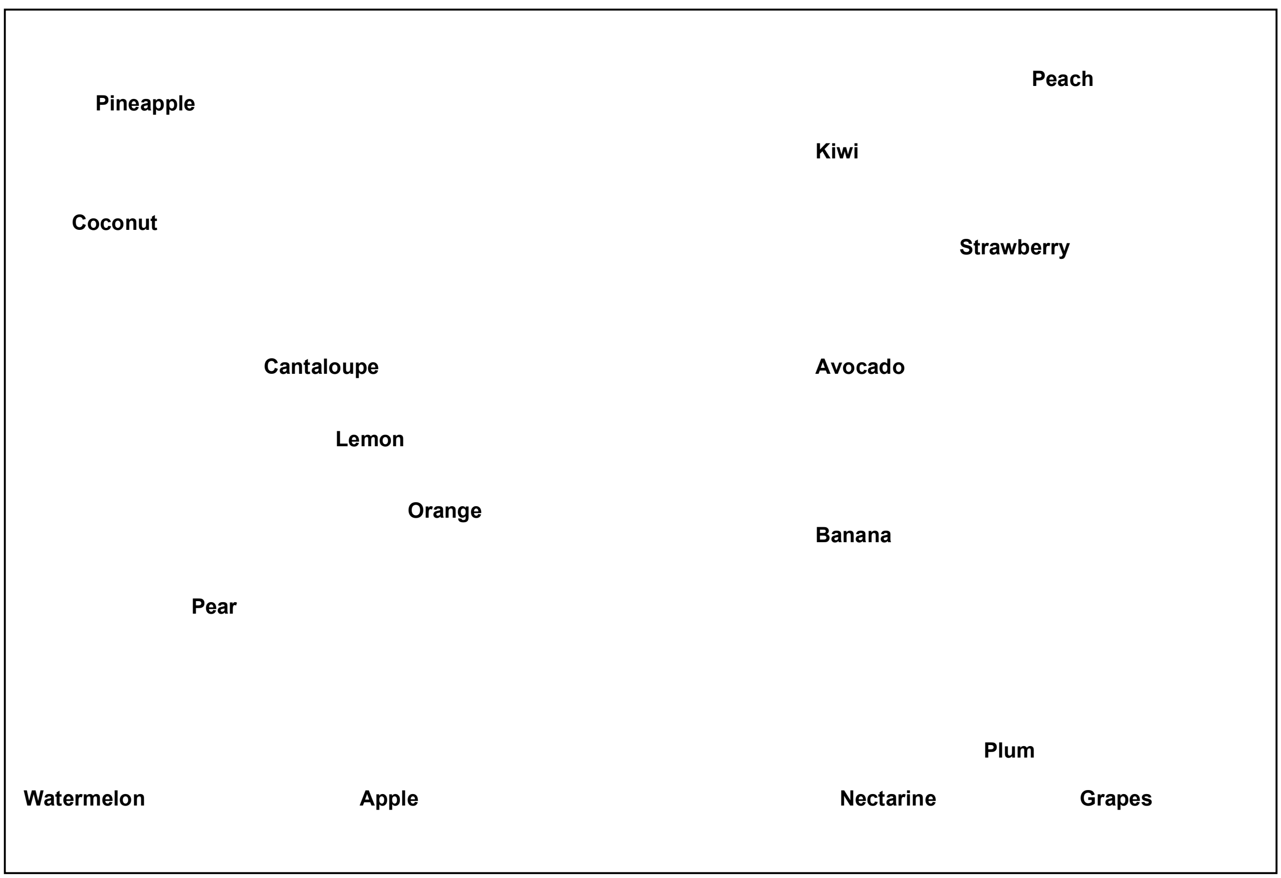
Correlations between pairs of fruits were calculated. A sample of these correlations appears below.

	Pineapple	Coconut	Strawberry
Pineapple	1.00	0.20	0.78
Coconut	0.20	1.00	0.43
Strawberry	0.78	0.43	1.00
These correlations are not the actual data from the study.			

Because higher correlations represent shorter "distances" between fruit, we calculate distances by taking 1 – r.

	Pineapple	Coconut	Strawberry
Pineapple	0	0.80	0.22
Coconut	0.80	0	0.57
Strawberry	0.22	0.57	0
Numbers represent one minus the correlation			

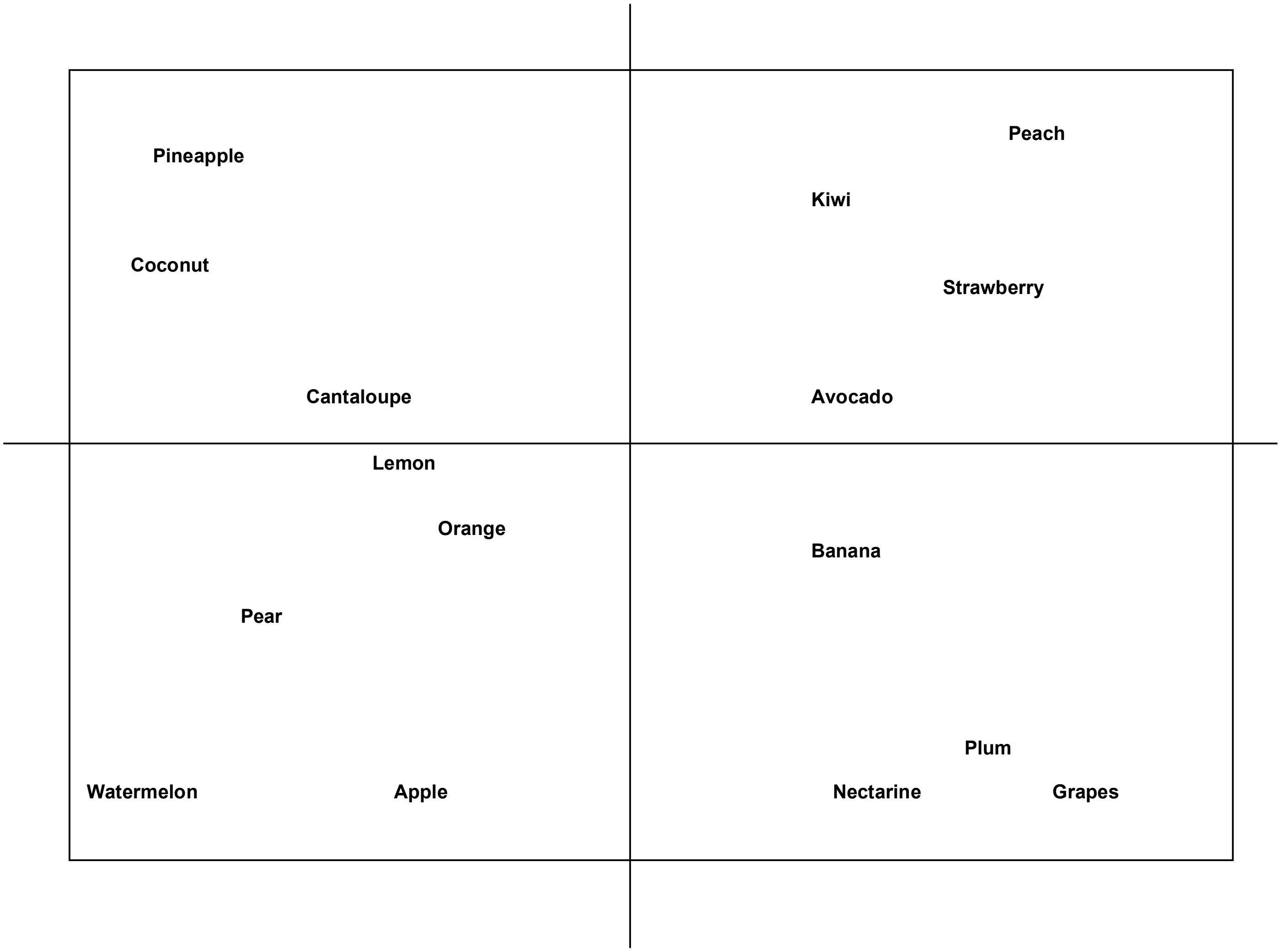
Working under the assumption that two dimensions underlie consumers' preferences for specific fruits, we have a computer create a map of these distances:



Take time to look for something systematic in the fruits. Why are some fruits grouped together? Why are other fruits far apart? What do the fruits at the top, right, bottom, and left have in common? What are the differences?

Remember that this process creates a map with correct distances, but incorrect orientation. There is no “true north” in an MDS map. We can sketch in a set of axes to aid in interpretation.

The map is again presented below. In my opinion, there are at least two axes that can be interpreted. Can you figure out what the axes represent?



As another example, let’s look at the results from a survey given to SAU alumni. In this survey, alumni were asked to rate their satisfaction with the preparation they received at SAU in 20 skills. Subjects selected responses from a 5-point Likert Scale (1 = not at all satisfied; 5 = extremely satisfied).

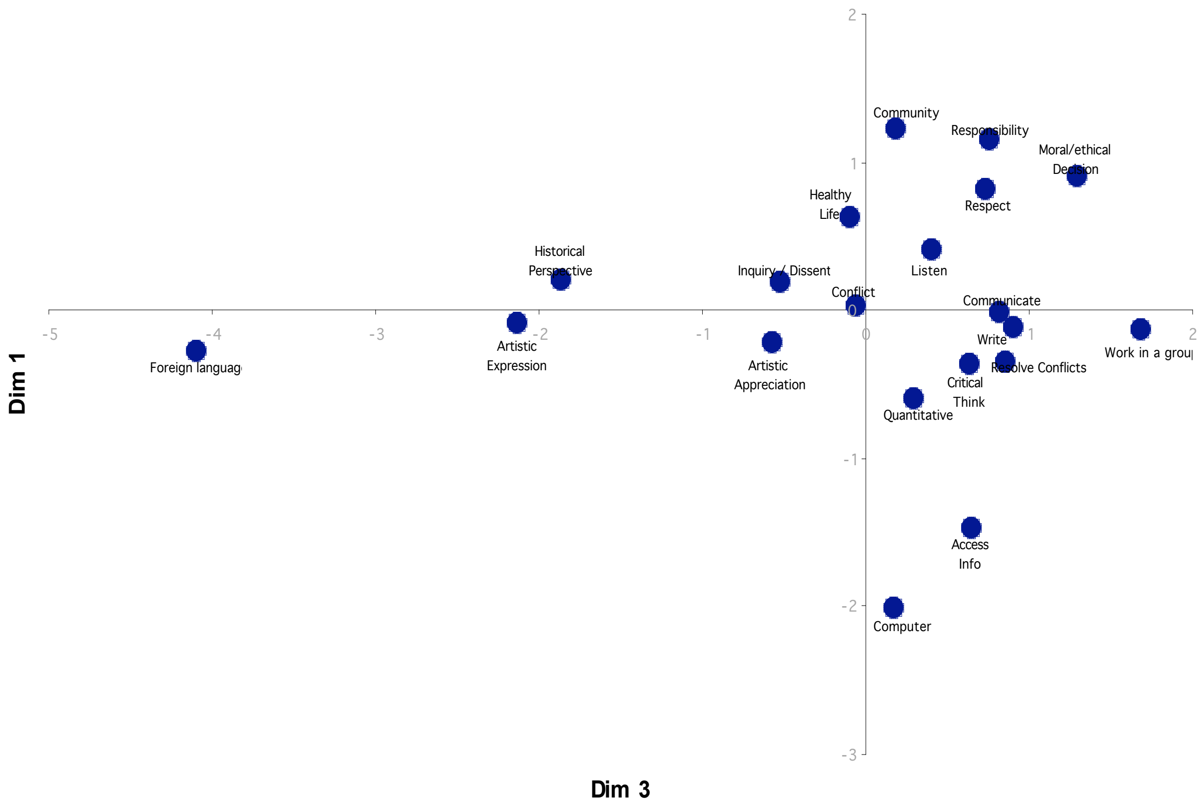
The skills included:

1. Communicate well orally
2. Listen effectively
3. Think critically
4. Solve problems effectively
5. Write effectively
6. Take responsibility for my actions
7. Make moral and ethical decisions
8. Use computer adequately
9. Resolve conflicts effectively
10. Locate appropriate sources of information
11. Respect individual differences
12. Work effectively in a group
13. Make health life decisions
14. Think quantitatively
15. Participate in the life of my community
16. Recognize freedom of inquiry allows for dissent
17. Appreciate artistic and other events
18. Place issues in historical perspective
19. Express self through an artistic medium
20. Communicate in a foreign language

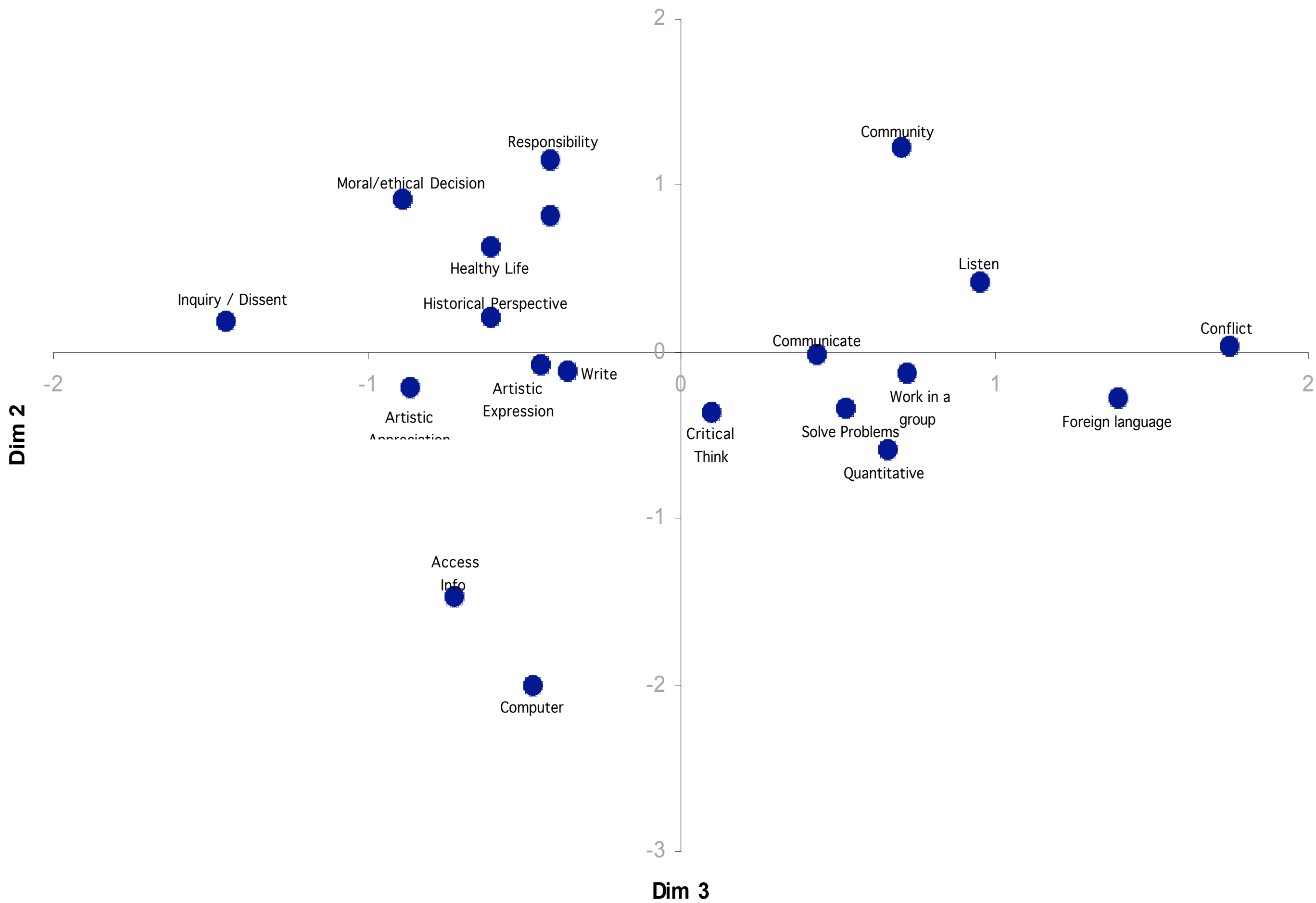
If a subject thought SAU prepared them well in communication skills, that subject would rate the skill a 5. If the subject thought SAU did not teach computer skills at all, that skill would receive a rating of 1.

The question to be answered was: What are the underlying reasons why some skills are rated higher than others?

The following 3-dimensional map was created. Can you interpret the axes?



Dim 1
Dim 3
Community
Responsibility
Moral/ethical Decision
Healthy Life
Respect
Listen
Historical Perspective
Inquiry / Dissent
Conflict
Communicate
Write
Work in a grou
Foreign languag
Artistic Expression
Artistic Appreciation
Resolve Conflicts
Critical Think
Quantitative
Access Info
Computer



Dim 2
Dim 3
Community
Responsibility
Moral/ethical Decision
Healthy Life
Listen
Inquiry / Dissent
Historical Perspective
Communicate
Conflict
Write
Artistic Expression
Artistic
Work in a group
Foreign language
Critical Think
Solve Problems
Quantitative
Access Info
Computer

8. Consider yet another example. 1,000 individuals were asked to rate ten different sodas on eight characteristics. The average ratings are displayed below:

	Coke	Coke Cl.	Diet Pepsi	Diet Slice	Diet 7-up	Dr Pepper	Pepsi	Slice	Tab	7-up
Fruity	5.79	6.49	5.8	2.91	4.29	4.03	5.73	1.38	5.22	2.86
Carbonation	3.42	3.89	4.87	5.66	4.93	4.36	3.14	5.18	5.24	3.89
Calories	4.68	5.57	3.36	3.47	3.63	5.4	4.61	4.84	3.8	4.5
Tart	3.32	4.24	5.01	6.08	6.22	4.47	2.71	3.73	5.35	3.52
Thirst	4.56	4.19	5.56	5.08	5.52	4.77	4.15	2.77	5.24	2.78
Popularity	3.35	2.21	4.05	5.86	6.31	5.1	2.24	5.63	5.35	3.98
Aftertaste	3.95	3.7	5.28	5.21	5.61	4.89	3.71	4.03	5.17	2.98
Pick-up	3.07	2.71	4.73	6.33	6.31	4.24	3.08	5.07	5.12	4.15

The data were entered into Stata and a metric multidimensional scaling analysis was conducted. The analysis found that 2 dimensions accounted for 92.84% of the variability among soda brands. The following table and graph display each soda brand's scores on those 2 dimensions. Try to sketch some orthogonal axes and interpret the results.

```
        Brand |          dim1          dim2
-------------+---------------------------
         Coke |        2.6514       -0.4366
 Coke_Classic |        3.5067       -0.8982
   Diet_Pepsi |       -0.2629       -2.1735
   Diet_Slice |       -3.6394       -0.1125
    Diet_7-up |       -3.3073       -1.3998
    Dr_Pepper |       -0.3478        0.2468
        Pepsi |        3.6278       -0.0039
        Slice |       -1.6579        3.5434
          Tab |       -1.4597       -1.5216
         7-up |        0.8892        2.7559
-----------------------------------------
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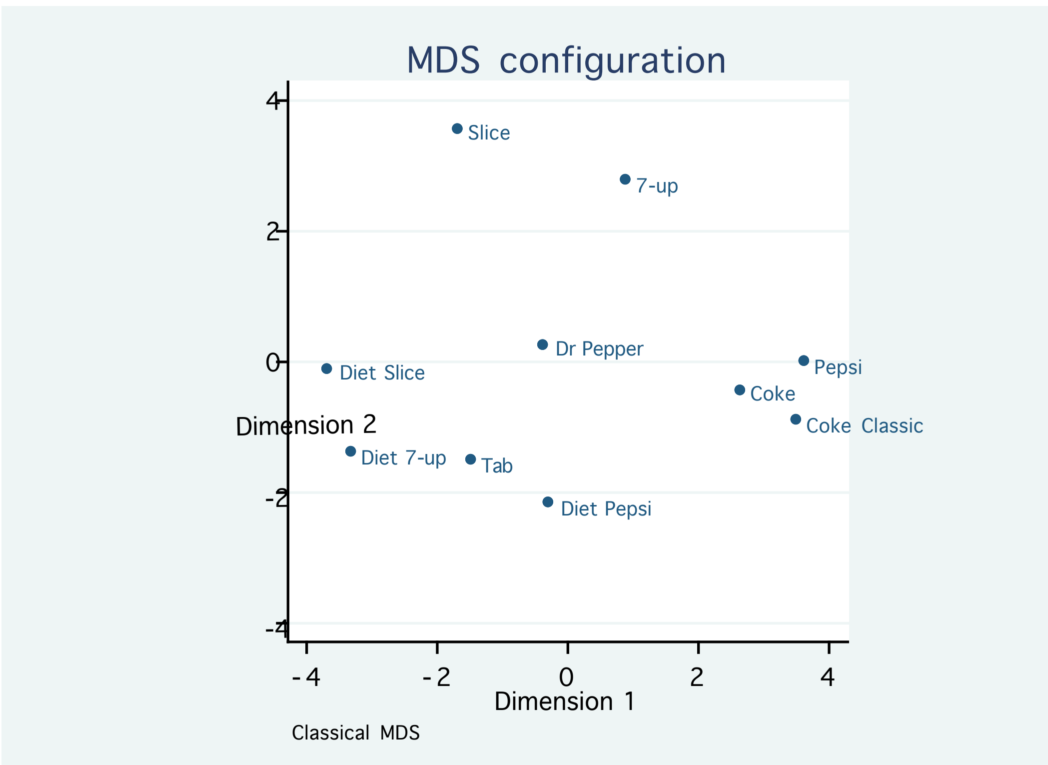
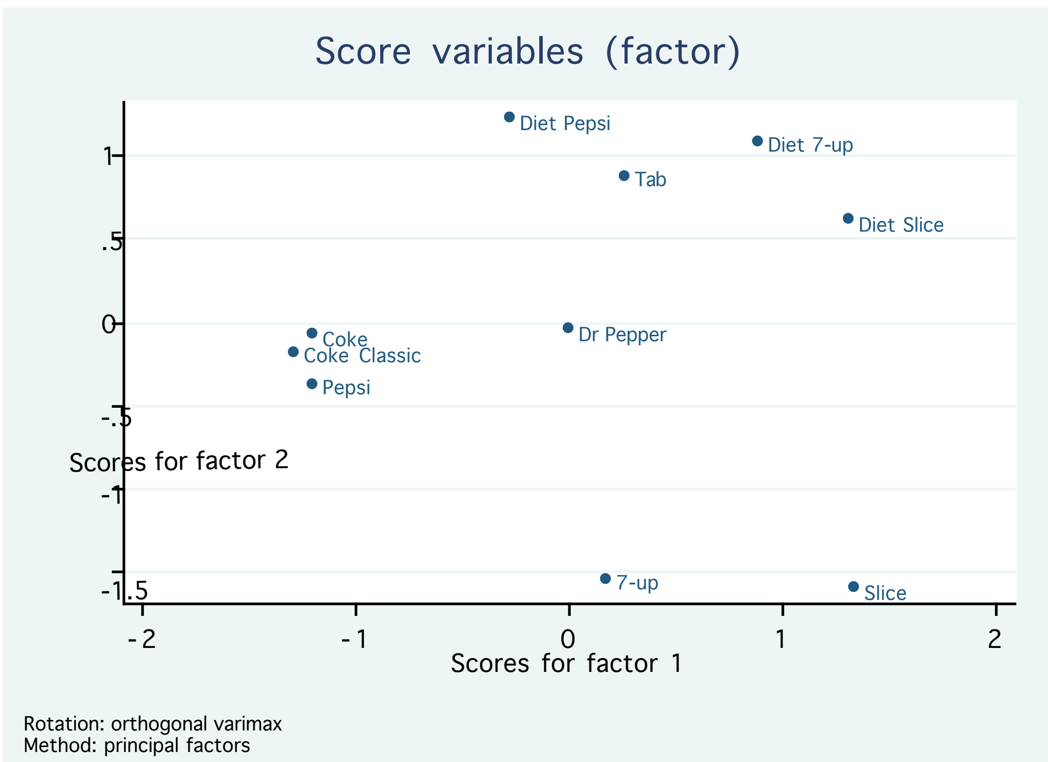


A factor analysis (with Varimax rotation) gives similar results. First, the variables are related to factors (factor loadings).

Variable	Factor1	Factor2	Uniqueness
Fruity	-0.8377	0.5133	0.0348
Carbonated	0.8526	0.3749	0.1325
Calories	-0.4581	-0.6237	0.4011
Tart	0.5958	0.7281	0.1149
Thirst	-0.0335	0.9847	0.0292
Popularity	0.9165	0.2501	0.0975
Aftertaste	0.4532	0.8489	0.0739
Pick_up	0.9082	0.3918	0.0216

Then each brand is given a factor score. The graph displays this information.



You can see this graph is just a rotation of the previous graph.