Exercise #1: Sugar levels in cereals (Follow-Up Tests: Conduct all possible pairwise comparisons)

Source	SS	df	MS	MSR
Shelf Effect	275.03	2	137.515	$\frac{137.515}{3.996} = 34.413$
Error	227.8	57	3.996	$F_{57}^{2} = 3.15$
Total	502.83	59		

		Sugar	
	N	Mean	SD
Shelf #1	20	4.80	2.138
Shelf #2	20	9.85	1.985
Shelf #3	20	6.10	1.865

Bonferroni Method: $t_{obs} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{MS_E\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where we control the family-wise alpha rate. We have $k = \frac{a(a-1)}{2}$ pairwise comparisons.

Comparison	Difference	Standard Error	t-critical	Confidence Interval	Significant?
$\overline{X}_1$ vs $\overline{X}_2$	$\overline{X}_1 - \overline{X}_2$	$\sqrt{MS_E\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$	$t_{critical} = t_{\frac{\alpha}{2k};(N-a)}$	$Difference \pm t(SE)$	Yes, if zero isn't in the CI
Shelf #1 vs #2	$4.8 - 9.85 = -5.05$	$\sqrt{3.996\left(\frac{2}{20}\right)} = 0.632$	$t_{\frac{.05}{6},(60-3)} = t_{0.008,57} = 2.45$	$5.05 \pm (2.45)(.632) = (-6.6, -3.5)$	Yes
Shelf #1 vs #3	$4.8 - 6.1 = -1.3$	0.632	2.45	(-2.84, 0.248)	No
Shelf #2 vs #3	$9.85 - 6.1 = 3.75$	0.632	2.45	(2.20, 5.30)	Yes

Our alpha was 0.008.
We conclude that Shelf #1 cereals have less sugar than Shelf #2 cereals and Shelf #3 cereals have less sugar than Shelf #2 cereals.

Tukey Method: $Q_{a,N-a,\alpha}\sqrt{\frac{\frac{MS_E}{2n_1n_2}}{n_1+n_2}}$ $\quad Q_{a,N-a,\alpha}\sqrt{\frac{\frac{MS_E}{2n_1n_2}}{n_1+n_2}}$

Comparison	Observed Difference	Standard Error	Q-value	HSD	Significant?
1 vs 2	-5.05	$\sqrt{\frac{\frac{3.996}{2(20)(20)}}{20+20}} = 0.447$	3.4	1.52	Yes
1 vs 3	-1.30	0.447	3.4	1.52	No
2 vs 3	3.75	0.447	3.4	1.52	Yes

Scheffe Method: Suppose we want to compare Shelves #1 and #3 to Shelf #2.

Contrast: $\psi = c_1\mu_1 + c_2\mu_2 + \ldots + c_a\mu_a$ where $c_1 + c_2 + \ldots + c_a = 0$

Our contrast of interest: $\psi = (c_1\mu_1 + c_3\mu_3) - c_2\mu_2$

So $(c_1 + c_3) - c_2 = 0$

Therefore, $(c_1 + c_3) = c_2$

We could make these weights anything we want as long as it satisfies the above condition

$\psi = (1\mu_1 + 1\mu_3) - 2\mu_2$ or $\psi = (\frac{1}{2}\mu_1 + \frac{1}{2}\mu_3) - 1\mu_2$

Our observed contrast: $\psi = \left[\frac{1}{2}(4.8) + \frac{1}{2}(6.1)\right] - 1(9.85) = -4.4$

Our standard error: $\sqrt{MS_E \sum \frac{c_a^2}{n_a}} = \sqrt{(3.996)\left[\frac{\left(\frac{1}{2}\right)^2}{20} + \frac{\left(\frac{1}{2}\right)^2}{20} + \frac{(-1)^2}{20}\right]} = 0.547$

Our observed test statistic is: $\frac{\psi}{SE} = \frac{-4.4}{0.547} = -8.04$

We compare this to: $\sqrt{(a-1)F_{N-a}^{a-1}} = \sqrt{(3-1)F_{57}^{21}} = \sqrt{2(3.15)} = 2.51$

Since our observed test statistic is bigger than this critical value, we reject the null hypothesis.

Exercise #6: Caffeine's effect on finger tapping speed (Follow-Up Tests: Conduct all possible pairwise comparisons)

Source	SS	df	MS	MSR
Caffeine Effect	61.4	2	30.7	6.181
Error	134.1	27	4.967	$F^2_{27} = 3.35$
Total	195.5	29		

	Taps		
	N	Mean	SD
0 mg	10	244.8	2.394
100 mg	10	246.4	2.066
200 mg	10	248.4	2.214

Bonferroni Method: $t_{obs} = \dfrac{\overline{X}_1 - \overline{X}_2}{\sqrt{MS_E\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where we control the family-wise alpha rate. We have $k = \dfrac{a(a-1)}{2}$ pairwise comparisons.

Comparison	Difference	Standard Error	t-critical	Confidence Interval	Significant?
$\overline{X}_1$ vs $\overline{X}_2$	$\overline{X}_1 - \overline{X}_2$	$\sqrt{MS_E\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$	$t_{critical} = t_{\frac{\alpha}{2k};(N-a)}$	$Difference \pm t(SE)$	Yes, if zero isn't in the CI
0mg vs 100mg	-1.6	$\sqrt{4.967\left(\frac{2}{20}\right)} = 0.997$	$t_{\frac{.05}{6},(60-3)} = t_{0.008,27} = 2.5$	(-4.1 , 0.89)	No
100mg vs 200mg	-2.0	0.997	2.5	(-4.5 , 0.49)	No
0mg vs 200mg	-3.6	0.997	2.5	(-6.1 , -1.1)	Yes

Our alpha was 0.008.
We conclude that 200mg of caffeine yielded significantly higher taps than 0mg of caffeine.

Tukey Method: $Q_{a,N-a,\alpha}\sqrt{\dfrac{\frac{MS_E}{2n_1n_2}}{n_1+n_2}}$ $\quad Q_{a,N-a,\alpha}\sqrt{\dfrac{\frac{MS_E}{2n_1n_2}}{n_1+n_2}}$

Comparison	Observed Difference	Standard Error	Q-value	HSD	Significant?
0mg vs 100mg	-1.6	$\sqrt{\dfrac{\frac{4.967}{2(10)(10)}}{10+10}} = 0.7047$	3.5	2.467	No
100mg vs 200mg	-2.0	0.7047	3.5	2.467	No
0mg vs 200mg	-3.6	0.7047	3.5	2.467	Yes

Scheffe Method: Suppose we want to compare caffeine (100 and 200mg) to no caffeine (0 mg).

Contrast: $\psi = c_1\mu_1 + c_2\mu_2 + \ldots + c_a\mu_a$ where $c_1 + c_2 + \ldots + c_a = 0$

Our contrast of interest: $\psi = (c_2\mu_2 + c_3\mu_3) - c_1\mu_1$

So $(c_2 + c_3) - c_1 = 0$

Therefore, $(c_2 + c_3) = c_1$

We could make these weights anything we want as long as it satisfies the above condition

$\psi = (1\mu_2 + 1\mu_3) - 2\mu_1$ or $\psi = (\frac{1}{2}\mu_2 + \frac{1}{2}\mu_3) - 1\mu_1$

Our observed contrast: $$\psi = \left[\frac{1}{2}(246.4) + \frac{1}{2}(248.4)\right] - 1(244.8) = 2.6$$

Our standard error: $$\sqrt{MS_E \sum \frac{c_a^2}{n_a}} = \sqrt{(4.967)\left[\frac{\left(\frac{1}{2}\right)^2}{10} + \frac{\left(\frac{1}{2}\right)^2}{10} + \frac{(-1)^2}{10}\right]} = 0.863$$

Our observed test statistic is: $$\frac{\psi}{SE} = \frac{2.6}{0.863} = 3.013$$

We compare this to: $$\sqrt{(a-1)F_{N-a}^{a-1}} = \sqrt{(3-1)F_{27}^{2}} = \sqrt{2(3.35)} = 2.588$$

Since our observed test statistic is bigger than this critical value, we reject the null hypothesis.