Six scenarios are presented on the next page. The subsequent pages display ANOVAs calculated by hand for each scenario.

Choose one of the scenarios and calculate:
(a) Two pairwise t-tests using the Bonferroni correction
(b) One test of a contrast using the Scheffe method. Pick a contrast that might be of interest to a researcher.

ANOVA Exercises

A sample of 20 different types of cereals was taken from each of three grocery store shelves (1, 2, and 3, counting from the floor). A summary of the sugar content (grams per serving) and dietary fiber (grams per serving) of the cereals is given below.

		Sugar		Fiber	
	N	Mean	SD	Mean	SD
Shelf #1	20	4.80	2.138	1.68	1.166
Shelf #2	20	9.85	1.985	0.95	1.162
Shelf #3	20	6.10	1.865	2.17	1.277

1. Test for significant differences in sugar content among the three shelves.
   a. Assess whether or not the data meet the assumptions necessary for an ANOVA.
   b. Construct an ANOVA summary table and find the observed mean square ratio.
   c. Compare the observed mean square ratio to its appropriate test statistic
   d. State your conclusion (explain what rejecting or retaining the null hypothesis means)
   e. According to your results, which shelf displays the cereals with the highest sugar content?
   f. Calculate the effect size ($\eta^2 = SS_A / SS_T$) and explain what it represents.

2. Test for significant differences in fiber content among the three shelves.
   a. Assess whether or not the data meet the assumptions necessary for an ANOVA.
   b. Construct an ANOVA summary table and find the observed mean square ratio.
   c. Compare the observed mean square ratio to its appropriate test statistic
   d. State your conclusion (explain what rejecting or retaining the null hypothesis means)
   e. According to your results, which shelf displays the cereals with the highest fiber content?
   f. Calculate the effect size ($\eta^2 = SS_A / SS_T$) and explain what it represents.

3. A factory has three production lines producing glass sheets that are all supposed to be of the same thickness. A quality inspector takes a random sample of n = 30 sheets from each production line and measures their thickness. The glass sheets from the first production line have a sample mean of 3.015 mm with a sample standard deviation of 0.107 mm. The sample mean and standard deviation of the second production line are 3.018 mm and 0.155 mm. The third production line produced glass with a mean of 2.996 mm and a standard deviation of 0.132 mm. What conclusions should the quality inspector draw? Calculate the effect size ($\eta^2 = SS_A / SS_T$) and explain what it represents.

4. The presence of harmful insects in farm fields is detected by erecting boards covered with a sticky material and then examining the insects trapped on the boards. To investigate which colors are most attractive to cereal leaf beetles, researchers placed six boards of each of four colors in a field of oats in July. Using the data in the following table, what conclusions can you draw? Calculate the effect size ($\eta^2 = SS_A / SS_T$) and explain what it represents.

	Number of Insects Trapped						Mean	Std. Dev.
Yellow	45	59	48	46	38	47	47.167	6.795
White	21	12	14	17	13	17	15.667	3.327
Green	37	32	15	25	39	41	31.500	9.915
Blue	16	11	20	21	14	7	14.833	5.345

5. Complete the following ANOVA summary tables. Conduct the significance test and calculate the effect size ($\eta^2 = SS_A / SS_T$) for each table.

Source	SS	df	MS	MSR
Groups		3	158.96	
Error		32	62.81	
Total				

Source	SS	df	MS	MSR
Groups		7		5.01
Error			3.62	
Total		29		

6. The effect of caffeine levels on performing a simple finger-tapping task was investigated. 30 male college students were trained in finger tapping and randomly assigned to receive either 0, 100, or 200 mg of caffeine. Two hours later, the students were asked to finger tap and the number of taps per minute was counted. What conclusions can you draw from this data (using SPSS as a class). If the differences among groups are statistically significant, are they also of practical significance?

Caffeine	Taps per minute	Mean	Std. Dev.
0 mg	242 245 244 248 247 248 242 244 246 242	244.8	2.394
100 mg	248 246 245 247 248 250 247 246 243 244	246.4	2.066
200 mg	246 248 250 252 248 250 246 248 245 250	248.4	2.214

Solutions to ANOVA exercises

NOTE: You can use an online ANOVA calculator at http://www.danielsoper.com/statcalc/calc43.aspx (google "ANOVA calculator")

1) Decide whether or not an ANOVA is appropriate.
   a) Are we comparing two or more group (treatment) means?
   b) Does our data meet the assumptions necessary to run an ANOVA?
      a. Are the groups independent?
      b. Are the observations within each group normally distributed? (Graph, p-p plot, or use Kolmogorov-Smirnov test)
      c. Homogeneity of variance assumption: Are the group/treatment variances equal?
         i. Use the F_{max} test to check for equal variances:

Compute: $F_{\max} = \dfrac{\sigma^2_{largest}}{\sigma^2_{smallest}}$ and compare to $F^{\,a}_{\max\, n-1}$ from the F_{max} table, where a = # of groups and n = observations per group.

If your calculated F_{max} < the F_{max} value from the table, you can procede.
If your calculated F_{max} > the F_{max} value from the table, we cannot conduct an ordinary ANOVA.

   c) State the null and alternate hypotheses: $H_0 : \mu_1 = \mu_2 = \ldots = \mu_a$ vs. *H_A: Not H_0*

or

$H_0 : \alpha_j = 0$ vs. $H_A : \alpha_j \neq 0$

   d) Calculate the sample means and standard deviations for each treatment: $\overline{X}_a = \dfrac{1}{n_a}\sum x_{ia}$ and $s_a = \sqrt{\dfrac{\sum\left(x_{ia} - \overline{X}_a\right)^2}{n_a - 1}}$

   e) Calculate the overall mean: $M = \overline{\overline{X}} = \dfrac{\sum n_a \overline{X}_a}{N}$

   f) Calculate sums of squares: $SS_A = \sum n_a\left(\overline{X}_a - M\right)^2$ $\qquad SS_E = \sum (n_a - 1)s_a^2$ $\qquad SS_T = SS_A + SS_E$

   g) Calculate degrees of freedom: $df_A = a - 1$ $\qquad df_E = N - a$ $\qquad df_T = N - 1$

   h) Fill-in the ANOVA summary table with your calculated SS and df values:

Source	Sums of Squares	Degrees of freedom	Mean Square	Mean Square Ratio
Between Groups (Treatment Effect)				
Within Groups (Error variance)				
Total				

   i) Calculate the mean squares: $MS_A = \dfrac{SS_A}{df_A}$ $\qquad MS_E = \dfrac{SS_E}{df_E}$

   j) Calculate the mean square ratio: $MSR = \dfrac{MS_A}{MS_E}$ and compare it to F^{a-1}_{N-a} from the F-tables.

If MSR > F^{a-1}_{N-a} we reject the null hypothesis. If MSE < F^{a-1}_{N-a} we retain the null hypothesis.

   k) Calculate the effect size: $\eta^2 = \dfrac{SS_A}{SS_T}$ (the proportion of total variance accounted for by the treatments)

   l) Write your conclusion and perform appropriate follow-up tests.

Exercise #1:

		Sugar	
	N	Mean	SD
Shelf #1	20	4.80	2.138
Shelf #2	20	9.85	1.985
Shelf #3	20	6.10	1.865

$H_0 : \alpha_j = 0$ (the three shelves have equal population means; no treatment effect)

$H_A : \alpha_j \neq 0$ (the treatment means are not all equal; a treatment effect exists)

$F_{\max} = \frac{\sigma^2_{largest}}{\sigma^2_{smallest}} = \frac{2.138^2}{1.865^2} = 1.314$ compared to $F^{3}_{\max 19} = 2.95$ (OK to move on)

$$M = \overline{\overline{X}} = \frac{(20)(4.8) + (20)(9.85) + (20)(6.10)}{60} = 6.9167$$

$$MS_A = \frac{SS_A}{df_A} = \frac{(20)(4.8 - 6.9167)^2 + (20)(9.85 - 6.9167)^2 + (20)(6.10 - 6.9167)^2}{3-1} = \frac{275.03}{2} = 137.515$$

$$MS_E = \frac{SS_E}{df_E} = \frac{(20-1)(2.138)^2 + (20-1)(1.985)^2 + (20-1)(1.865)^2}{60-3} = \frac{227.8}{57} = 3.996$$

Source	SS	df	MS	MSR
Shelf Effect	275.03	2	137.515	$\frac{137.515}{3.996} = 34.413$
Error	227.8	57	3.996	$F^2_{57} = 3.15$
Total	502.83	59		

Since our calculated (observed) ratio is bigger than the critical F-value, we reject the null hypothesis.

$$\eta^2 = \frac{275.03}{502.83} = 0.547$$

54.7% of the variance in sugar is accounted for by shelf location.

Exercise #2:

		Fiber	
	N	Mean	SD
Shelf #1	20	1.68	1.166
Shelf #2	20	0.95	1.162
Shelf #3	20	2.17	1.277

$H_0 : \alpha_j = 0$ (the three shelves have equal population means; no treatment effect)

$H_A : \alpha_j \neq 0$ (the treatment means are not all equal; a treatment effect exists)

$F_{\max} = \frac{\sigma^2_{largest}}{\sigma^2_{smallest}} = \frac{1.277^2}{1.166^2} = 1.208$ compared to $F^{3}_{\max 19} = 2.95$ (OK to move on)

$$M = \overline{\overline{X}} = \frac{(20)(1.68) + (20)(0.95) + (20)(2.17)}{60} = 1.6$$

$$MS_A = \frac{SS_A}{df_A} = \frac{(20)(1.68 - 1.6)^2 + (20)(0.95 - 1.6)^2 + (20)(2.17 - 1.6)^2}{3-1} = \frac{15.076}{2} = 7.538$$

$$MS_E = \frac{SS_E}{df_E} = \frac{(20-1)(1.166)^2 + (20-1)(1.162)^2 + (20-1)(1.277)^2}{60-3} = \frac{82.47}{57} = 1.447$$

Source	SS	df	MS	MSR
Shelf Effect	15.076	2	7.538	$\frac{7.538}{1.447} = 5.21$
Error	82.47	57	1.447	$F^2_{57} = 3.15$
Total	97.546	59		

Since our calculated (observed) ratio is bigger than the critical F-value, we reject the null hypothesis.

$$\eta^2 = \frac{15.076}{97.546} = 0.15$$

15% of the variance in fiber is accounted for by shelf location.
Is this of practical significance?

Exercise #3:

		Thickness	
	N	Mean	SD
Group 1	30	3.015	0.107
Group 2	30	3.018	0.155
Group 3	30	2.996	0.132

$H_0 : \alpha_j = 0$ (the three production lines produce glass of the same thickness)

$H_A : \alpha_j \neq 0$ (the production line effects the thickness of glass)

$F_{\max} = \frac{\sigma^2_{largest}}{\sigma^2_{smallest}} = \frac{0.155^2}{0.107^2} = 2.098$ compared to $F^{3}_{\max 29} = 2.07$ (We shouldn't continue)

$$M = \overline{\overline{X}} = \frac{(30)(3.015) + (30)(3.018) + (30)(2.996)}{90} = 3.01$$

$$MS_A = \frac{SS_A}{df_A} = \frac{(30)(3.015-3.01)^2 + (30)(3.018-3.01)^2 + (30)(2.996-3.01)^2}{3-1} = \frac{0.00855}{2} = 0.004275$$

$$MS_E = \frac{SS_E}{df_E} = \frac{(30-1)(.107)^2 + (30-1)(.155)^2 + (30-1)(.132)^2}{90-3} = \frac{1.534}{87} = 0.0176$$

Source	SS	df	MS	MSR
Line	.0086	2	.004275	$\frac{.004275}{.0176} = 0.243$
Error	1.534	87	0.0176	$F^2_{87} = 3.09$
Total	1.5426	89		

Since our calculated (observed) ratio is smaller than the critical F-value, we retain the null hypothesis.

$$\eta^2 = \frac{.0086}{1.534} = 0.0056$$

0.56% of the variance in glass thickness is due to production line differences. You can see why there is no significant group effect.

Exercise #4:

		Insects	
	N	Mean	SD
Yellow	6	47.167	6.795
White	6	15.667	3.327
Green	6	31.5	9.915
Blue	6	14.833	5.345

$H_0 : \alpha_j = 0$ (the colors make no difference)

$H_A : \alpha_j \neq 0$ (the treatment means are not all equal; a treatment effect exists)

$F_{\max} = \frac{9.915^2}{3.327^2} = 9.19$ compared to $F^{4}_{\max 5} = 13.7$ (OK to move on)

$$M = \overline{\overline{X}} = \frac{6[47.167 + 15.667 + 31.5 + 14.833]}{24} = 27.29$$

$$MS_A = \frac{SS_A}{df_A} = \frac{6\left[(47.167-27.29)^2 + (15.667-27.29)^2 + (31.5-27.29)^2 + (14.833-27.29)^2\right]}{4-1} = \frac{4218.54}{3} = 1406.18$$

$$MS_E = \frac{SS_E}{df_E} = \frac{(20-1)[6.795^2 + 3.327^2 + 9.915^2 + 5.345^2]}{24-4} = \frac{920.586}{20} = 46.03$$

Source	SS	df	MS	MSR
Color	4218.54	3	1406.18	$\frac{1406.16}{46.03} = 30.55$
Error	920.586	20	46.03	$F^3_{20} = 3.03$
Total	5139.126	23		

Since our calculated (observed) ratio is bigger than the critical F-value, we reject the null hypothesis.

$$\eta^2 = \frac{4218.54}{5139.126} = 0.82$$

82% of the variance in insects captured is accounted for by color.

Exercise #5:

Source	SS	df	MS	MSR
Groups	**476.88**	3	158.96	**2.53**
Error	**2009.92**	32	62.81	
Total	**2486.8**	**35**		

Source	SS	df	MS	MSR
Groups	**126.9534**	7	**18.1362**	5.01
Error	**79.64**	**22**	3.62	
Total	**206.5934**	29		

$$\eta^2 = \frac{476.88}{2486.8} = 0.19 \qquad \eta^2 = \frac{126.9534}{79.64} = 0.6145$$

Exercise #6:

Caffeine	Taps per minute	Mean	Std. Dev.
0 mg	242 245 244 248 247 248 242 244 246 242	244.8	2.394
100 mg	248 246 245 247 248 250 247 246 243 244	246.4	2.066
200 mg	246 248 250 252 248 250 246 248 245 250	248.4	2.214

Test of Homogeneity of Variances

TAPS

Levene Statistic	df1	df2	Sig.
.292	2	27	.749

ANOVA

TAPS

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	61.400	2	30.700	6.181	.006
Within Groups	134.100	27	4.967		
Total	195.500	29			

Multiple Comparisons

Dependent Variable: TAPS

	(I) CAFFENE	(J) CAFFENE	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Tukey HSD	0 mg	100 mg	-1.60	.997	.261	-4.07	.87
		200 mg	-3.50*	.997	.004	-5.97	-1.03
	100 mg	0 mg	1.60	.997	.261	-.87	4.07
		200 mg	-1.90	.997	.156	-4.37	.57
	200 mg	0 mg	3.50*	.997	.004	1.03	5.97
		100 mg	1.90	.997	.156	-.57	4.37
Bonferroni	0 mg	100 mg	-1.60	.997	.360	-4.14	.94
		200 mg	-3.50*	.997	.005	-6.04	-.96
	100 mg	0 mg	1.60	.997	.360	-.94	4.14
		200 mg	-1.90	.997	.202	-4.44	.64
	200 mg	0 mg	3.50*	.997	.005	.96	6.04
		100 mg	1.90	.997	.202	-.64	4.44

*. The mean difference is significant at the .05 level.